1. (30 points) Let

\[ IS = \{ (G, k) \mid \text{graph } G \text{ has an independent set of size } k \} \]

(a) (10 points) Show that \( \text{CNF-SAT} \leq IS \). Do not use \&\&\; CLIQUE as an intermediary. Explain why your reduction works.

(b) (10 points) On the formula:

\[ (x_1 \lor x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2) \land \neg x_3 \]

what \((G, k)\) does your reduction produce? Draw the graph.

(c) (10 points) On the formula:

\[ (x_1 \lor x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2) \land x_4 \]

what \((G, k)\) does your reduction produce? Draw the graph.

GO TO THE NEXT PAGE!!!!!!!!!!!!!!!!!!!!!
2. (35 points) Let

\[
\text{CLIQ} = \{(G, k) \mid \text{graph } G \text{ has a clique of size } k\}
\]

Let

\(\text{SCLIQ}\) be the FUNCTION that will, on input \(G\), output the SIZE of the largest clique.

Let

\(\text{FINDCLIQ}\) be the FUNCTION that will, on input \(G\), output both the SIZE of the largest clique, and SOME clique of that size (that is, a list of vertices that form a clique of max size).

(a) (17 points) Show that if \(\text{CLIQ} \in P\) then \(\text{SCLIQ}\) can be computed in polynomial time. (THINK ABOUT but don’t hand in: Your algorithm made several calls to the \(\text{CLIQ}\) TM. How many? Assuming \(P \neq NP\), is there a way to do with this with 18 calls to \(\text{CLIQ}\)?)

(b) (18 points) Show that if \(\text{CLIQ} \in P\) then \(\text{FINDCLIQ}\) can be computed in polynomial time. (THINK ABOUT but don’t hand in: Your algorithm made several calls to the \(\text{CLIQ}\) TM. How many? Assuming \(P \neq NP\), is there a way to do with this with 18 calls to \(\text{CLIQ}\)?)

3. (35 points) Let

\[
\text{3COL} = \{G : \text{graph } G \text{ is 3-colorable}\}.
\]

Show that \(3\text{COL} \leq SAT\). Give an explicit reduction and explain why it works.

(HINT: Let \(G\) have vertices \(1, \ldots, n\). The variables are, for \(1 \leq i \leq 3\), \(1 \leq j \leq n\), \(x_{ij}\). The variable \(x_{ij}\) is intended to be TRUE if Vertex \(j\) is colored \(i\), and FALSE if not.)