**Homework 1 Morally Due Feb 11**
**WARNING: THIS HW IS TWO PAGES LONG!!!!!!!!!!!!!!!!!!!!**

1. (0 points, but if you actually miss the midterm without telling Dr. Gasarch ahead of time, you will lose 100 points on this homework) When will the midterm be (give date and time)? When will the final be (give date and time)? By when do you have to tell Dr. Gasarch that you cannot make the midterm?

2. (0 points, but if you miss emails related to the course, you can't complain) If you are not getting emails that the class gets, then email Saadiq, Josh, and Dr. Gasarch as soon as possible. You should have gotten an email saying this homework was posted.

3. (35 points) The alphabet is $\{0, \ldots, 9\}$. We interpret the input as a base 10 natural number, read *right to left*. So the number 29139 will be read 9-3-1-9-2.

   (a) (10 points) Compute
   $10^0 \pmod{13}$
   $10^1 \pmod{13}$
   $10^2 \pmod{13}$
   etc. until you spot a pattern. What is the pattern?

   (b) (25 points) Give a DFA classifier that determines, given a number in base 10, what that number is congruent to mod 13. Include

   - $Q$, the state set
   - $\delta$, the transition function, **give a table not a diagram!**, the table should state how $\delta$ operates given the running sum and the position
   - For each $0 \le a \le 12$, the set $F_a$ of states that a number congruent to $a \pmod{13}$ will end up in.
   - (We're not asking you for the alphabet because we know it's $\{0, ..., 9\}$).

   **SOLUTION**

(a) Here are the powers of 10 (mod 13).

| $i$ | $10^i \pmod{13}$ |
|---|---|
| 0 | 1 |
| 1 | 10 |
| 2 | 9 |
| 3 | 12 |
| 4 | 3 |
| 5 | 4 |
| 6 | 1 |
| 7 | 10 |
| 8 | 9 |
| 9 | 12 |
| 10 | 3 |
| 11 | 4 |

So the pattern is $\{1, 10, 9, 12, 3, 4\}$

(b) The following gives the symbols of the DFA.

- $Q = \{0, ..., 12\} \times \{0, ..., 5\}$. The first number keeps track of the weighted sum mod 13 and the second number keeps track of the place of the next character in the string mod 6.
- $s = (0, 0)$
- $\delta((a, 0), i) = (a + 1 \times i \pmod{13}, 1)$
- $\delta((a, 1), i) = (a + 10 \times i \pmod{13}, 2)$
- $\delta((a, 2), i) = (a + 9 \times i \pmod{13}, 3)$
- $\delta((a, 3), i) = (a + 12 \times i \pmod{13}, 4)$
- $\delta((a, 4), i) = (a + 3 \times i \pmod{13}, 5)$
- $\delta((a, 5), i) = (a + 4 \times i \pmod{13}, 0)$
- For $0 \leq a \leq 13$:
  $F_a = \{(a, j) \mid 0 \leq j \leq 5\}$

**Note:** $Q$ was worth 9 points, $\delta$ was worth 8 points, and $F_a$ was worth 8 points. Some students forgot to include the final states or included the incorrect final states.

**END OF SOLUTION More Homework on next page!**

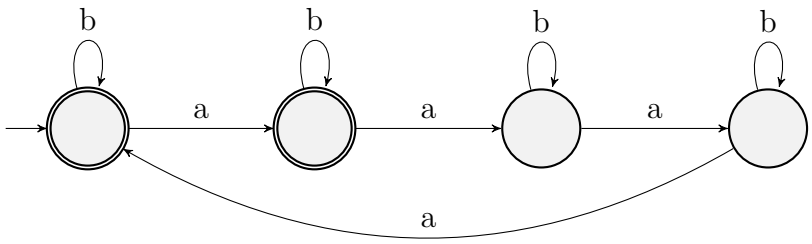
4. (35 points) We define $\#_a(w)$ to be the number of $a$'s in $w$. The alphabet is $\{a, b\}$.

   (a) (15 points) Draw a DFA for $\{w \mid \#_a(w) \equiv 0, 1 \pmod 4\}$. How many states does this DFA have?

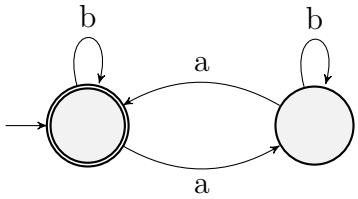
   (b) (20 points) Draw a DFA for $\{w \mid \#_a(w) \equiv 0, 2 \pmod 4\}$. How many states does this DFA have? (Hint: this should be less states than the prior part.)

**SOLUTION**

(a) This is the DFA. It has 4 states.




(b) This is the DFA. It has 2 states.




**END OF SOLUTION**

5. (30 points) The alphabet for this problem is $\{a, b\}$.

   (a) (7 points) Draw an NFA for the language $\Sigma^* aa \Sigma^*$. Try to make the number of states as small as possible. How many states does it have? **For your own benefit do an NFA for** $\Sigma^* a^3 \Sigma^*$.

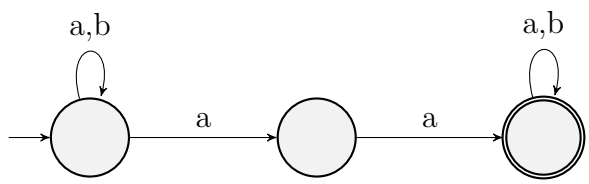
   (b) (7 points) Draw a DFA for the language $\Sigma^* aa \Sigma^*$. Use the NFA from part (a). Try to make the number of states as small as possible. How many states does it have? **For your own benefit do a DFA for** $\Sigma^* a^3 \Sigma^*$.

(c) (8 points) Draw an NFA for the language $\Sigma^* a^n \Sigma^*$. Try to make the number of states as small as possible. Use "..." to specify a certain number of states. How many states does it have as a function of $n$?

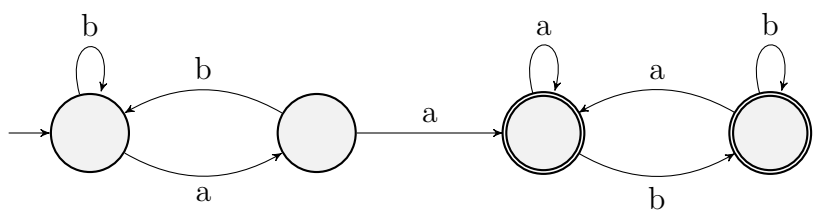
(d) (8 points) Draw a DFA for the language $\Sigma^* a^n \Sigma^*$. Use the NFA from part (e). Try to make the number of states as small as possible. Use "..." to specify a certain number of states. How many states does it have as a function of $n$?

**SOLUTION**

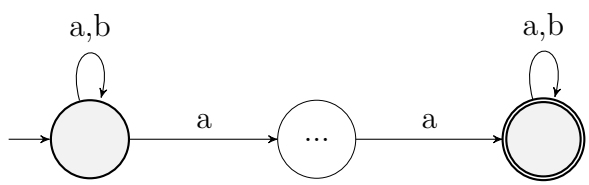
(a) This is the NFA. It has 3 states.
**Note:** there are two states with an $a, b$ loop to themselves. This can be written using $\Sigma$, which is fine. However, some of the students who we think did this, their sigmas looked like epsilons. We gave them the benefit of the doubt but in the future make sure your sigmas look like sigmas and your epsilons look like epsilons.




(b) This is the DFA. It has 4 states.

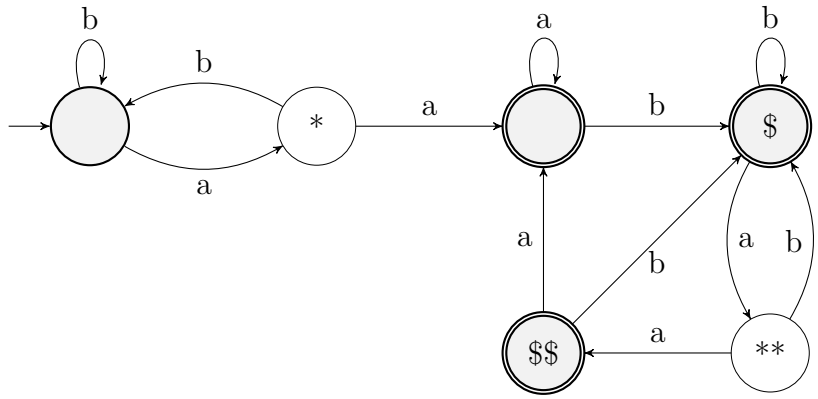



(c) This is the NFA. The "..." state represents $n-1$ states with "a" transitions going right. It has $n+1$ states.




(d) This is the NFA. The "*" state represents $n-1$ states with "a" transitions going right and with "b" transitions going to the start

state. The "**" state represents $n-3$ states with "a" transitions going right to the "$$" state and with "b" transitions going to the "$" state. In total, there are $2n$ states.




**END OF SOLUTION**