Homework 2 Morally Due Feb 18 at 11:00 AM
THIS HOMEWORK IS THREE PAGES LONG!!!!!!

1. (0 points, but if you actually miss the midterm without telling Dr. Gasarch ahead of time, you will lose 100 points on this homework) When will the midterm be (give date and time)? When will the final be (give date and time)? By when do you have to tell Dr. Gasarch that you cannot make the midterm?

2. (0 points, but if you miss emails related to the course, you can't complain) If you are not getting emails that the class gets, or if you are not on Piazza, then email Saadiq, Josh, and Dr. Gasarch as soon as possible.

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3. (40 points) The alphabet is $\{a\}$. For p prime let

$$L_p = \{a^i : i \equiv 0 \pmod{p}\}.$$

(a) (10 points) Describe a DFA for $L_2 \cup L_3 \cup L_5 \cup L_7$ by giving (Q, δ, s, F). δ should be described by a table. How many states does the DFA have? How many final states does it have? Hint: the DFA has a short description.

(b) (10 points) Describe an NFA for $L_2 \cup L_3 \cup L_5 \cup L_7$ by giving (Q, Δ, s, F). Δ should be described by a table. How many states does the NFA have? How many final states does it have? Hint: the NFA has a short description. Try to make the NFA have less states than the DFA. You might fail.

(c) (10 points) Describe a DFA for $L_2 \cap L_3 \cap L_5 \cap L_7$ by giving (Q, δ, s, F). δ should be described by a table. How many states does the DFA have? How many final states does it have? Hint: the DFA has a short description.

(d) (10 points) Describe an NFA for $L_2 \cap L_3 \cap L_5 \cap L_7$ by giving (Q, Δ, s, F). Δ should be described by a table. How many states does the NFA have? How many final states does it have? Hint: the NFA has a short description. Try to make the NFA have less states than the DFA. You might fail.

(e) (0 points) Think about: (1) are the DFAs in parts (a) and (c) optimal? (2) What about DFAs and NFAs for $L_2 \cup L_3 \cup L_5 \cup \cdots \cup L_{p_k}$ where p_k is the kth prime? (3) What about DFAs and NFAs for $L_2 \cap L_3 \cap L_5 \cap \cdots \cap L_{p_k}$ where p_k is the kth prime?

SOLUTION

(a) $Q = \{(i, j, k, l) : 0 \le i \le 1 \wedge 0 \le j \le 2 \wedge 0 \le k \le 4 \wedge 0 \le l \le 6\}$
$\delta((i, j, k, l), a) =$
$$(i + 1 \pmod 2, j + 1 \pmod 3, k + 1 \pmod 5, l + 1 \pmod 7)$$

$s = (0, 0, 0, 0)$
The final states are

$$F = \{(i, j, k, l) \in Q : i = 0 \vee j = 0 \vee k = 0 \vee l = 0\}$$

The DFA has a state for every combination of congruence classes mod $2, 3, 5, 7$ so it has $2 \cdot 3 \cdot 5 \cdot 7 = 210$ states. We now count the final states by counting the number of non-final states! The non-final states are

$$\overline{F} = \{(1, j, k, l) : 1 \leq j \leq 2 \wedge 1 \leq k \leq 4 \wedge 1 \leq l \leq 6\}$$

We see that $|\overline{F}| = 1 \cdot 2 \cdot 4 \cdot 6 = 48$ states that have no 0s, so $|F| = |Q| - |\overline{F}| = 210 - 48 = 162$.

(b) $Q = Q_2 \cup Q_3 \cup Q_5 \cup Q_7 \cup \{s\}$ where Q_i is the set of states for the DFA that accepts $L_i = \{a^j : j \equiv 0 \pmod{i}\}$ and s is the start state.
So we have $Q_i = \{(i, k) : 0 \leq k \leq i - 1\}$.
The NFA then has $2 + 3 + 5 + 7 + 1 = 18$ states.
$\Delta(s, \varepsilon) = \{(2, 0), (3, 0), (5, 0), (7, 0)\}$
$(\forall i \in \{2, 3, 5, 7\})\ (\forall 0 \leq j \leq i - 1)\ \Delta((i, j), a) = \{(i, j \pmod{i})\}$
The final states are

$$F = \{(2, 0), (3, 0), (5, 0), (7, 0)\}$$

so there are 4 final states.

(c) $Q = \{(i, j, k, l) : 0 \leq i \leq 1 \wedge 0 \leq j \leq 2 \wedge 0 \leq k \leq 4 \wedge 0 \leq l \leq 6\}$
$\delta((i, j, k, l), a) =$
$$(i + 1 \pmod{2}, j + 1 \pmod{3}, k + 1 \pmod{5}, l + 1 \pmod{7})$$

$s = (0, 0, 0, 0)$
The DFA has a state for every combination of congruence classes mod $2, 3, 5, 7$ so it has $2 \cdot 3 \cdot 5 \cdot 7 = 210$ states. There is only one final state

$$F = \{s\}.$$

(d) $Q = \{(i, j, k, l) : 0 \leq i \leq 1 \wedge 0 \leq j \leq 2 \wedge 0 \leq k \leq 4 \wedge 0 \leq l \leq 6\}$
$\Delta((i, j, k, l), a) =$
$$\{(i+1 \pmod{2}, j+1 \pmod{3}, k+1 \pmod{5}, l+1 \pmod{7})\}$$

$s = (0, 0, 0, 0)$
The NFA has a state for every combination of congruence classes

mod $2, 3, 5, 7$ so it has $2 \cdot 3 \cdot 5 \cdot 7 = 210$ states. There is only one final state

$$F = \{s\}.$$

END OF SOLUTION

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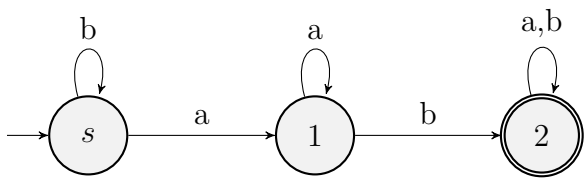
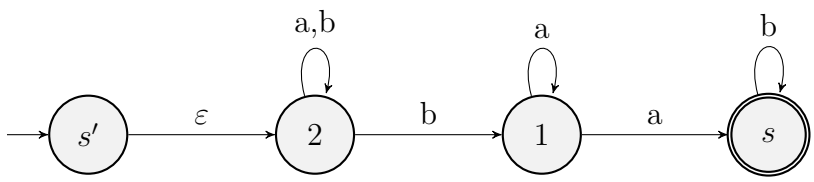
4. (30 points) Let w^R be the reverse of w and let $L^R = \{w : w^R \in L\}$. Prove that if L is regular then L^R is regular. Fill in the following: If the DFA for L has n states, then the DFA for L^R has $XXX(n)$ states. What is XXX?

SOLUTION

Let $M = (Q, \Sigma, \delta, s, F)$ be the DFA that accepts L.
Let $M^R = (Q \cup \{s'\}, \Sigma \cup \{\varepsilon\}, \Delta, s', \{s\})$ and $s' \notin Q$.
$\forall p \in Q, \sigma \in \Sigma$, $\Delta(p, \sigma) = \{q \in Q : \delta(q, \sigma) = p\}$
$\Delta(s', \varepsilon) = F$
The intuition is that if there is a path from p to q in M, then there must be a path from q to p in M^R.
As an example, let $L = \Sigma^* ab \Sigma^*$. The DFA that accepts L is



The NFA that accepts L^R is



We omit the DFA constructed from this NFA. If the DFA for L has n states, then the DFA for L^R has 2^{n+1} states.

END OF SOLUTION

5. (30 points) A SAADIQ-NFA is an NFA that has no ε-transitions. Give a procedure that takes an NFA and produces an equivalent SAADIQ-NFA. (Note - be careful. A sequence of ε-transitions can go through many states.)

SOLUTION

Let L be accepted by NFA $(Q, \Sigma, \Delta, s, F)$.

The equivalent SAADIQ-NFA is $(2^Q, \Sigma, \Delta', \{s\}, F')$. where

$\Delta'(A, \sigma)$ is the set of ALL states you can get to from any $q \in A \subseteq Q$ using any string of the form $\varepsilon^i \sigma \varepsilon^j$ in the old NFA.

F' is the set of all A such that $A \cup F \neq \emptyset$.

END OF SOLUTION