Solution to Hw 03 Problems 4 and 5

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Hw 03 Problem 4

1. Give an algorithm that does the following.
   Input: α a regular expression
   Output: β a regular expression for the complement of the language represented by α

   You may use any procedure described in class (e.g. procedures for intersection, union, complementation, powerset construction, equivalences between NFAs and DFAs, conversion from NFA to regular expression).
2. Think about: if α is of length n, how long is β?

Hw 03 Problem 4 Solution

SOLUTION

1. Input α ($|\alpha|$ is of length n)
2. Convert *alpha* to NFA N (N has $O(n)$ states)
3. Convert N to DFA M using powerset construction (M has $O(2^n)$ states)
4. Swap final and nonfinal states of M to get M'. 5. Convert M' to a regular expression β via $R(i, j, k)$ method ($|\beta|$ is $O(2^{2^n})$)

END OF SOLUTION

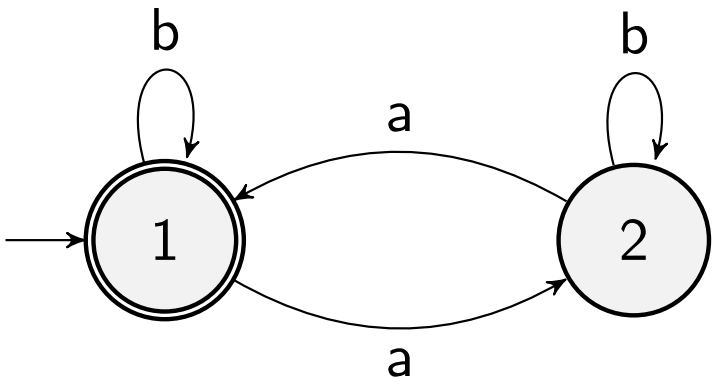
Hw 03 Problem 5

Let $L = \{w : \#_a(w) \equiv 0 \pmod 2\}$ and the alphabet is $\{a, b\}$.

1. Draw the DFA that accepts this language.
2. Use the $R(i, j, k)$ method to find the regex for this language. YOU MUST USE THIS METHOD EXACTLY AS GIVEN.
3. Determine the regex for L WITHOUT using the $R(i, j, k)$ method. It must be a SIMPLE regex. If we cannot understand your regex, YOU WILL GET NO CREDIT.

Hw 03 Problem 5 Solution Part 1

The following is the DFA for $L = \{w : \#_a(w) \equiv 0 \pmod 2\}$.



Hw 03 Problem 5 Solution Part 2

We begin with

$$\bigcup_{f \in F} R(1, f, n) = R(1, 1, 2) = R(1, 1, 1) \cup R(1, 2, 1) R(2, 2, 1)^* R(2, 1, 1)$$

so we ONLY want $R(1, 1, 2)$. We have

$$R(1, 1, 0) = \{\{e\}\} \cup \{b\} = \{\{e\}, b\}$$

$$R(1, 2, 0) = \{a\}$$

$$R(2, 1, 0) = \{a\}$$

$$R(2, 2, 0) = \{\{e\}\} \cup \{b\} = \{\{e\}, b\}$$

Hw 03 Problem 5 Solution Part 2 Continued

$$R(1,1,1) = R(1,1,0) \cup R(1,1,0)R(1,1,0)^* R(1,1,0) =$$

$$\{\{e\}, b\} \cup \{\{e\}, b\}\{\{e\}, b\}^* \{\{e\}, b\} = b^*$$

Hw 03 Problem 5 Solution Part 2 Continued

$$R(1,1,1) = R(1,1,0) \cup R(1,1,0)R(1,1,0)^* R(1,1,0) =$$
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$$R(1,2,1) = R(1,2,0) \cup R(1,1,0)R(1,1,0)^* R(1,2,0) =$$
$$\{a\} \cup \{\{e\}, b\}\{\{e\}, b\}^* a = b^* a$$

Hw 03 Problem 5 Solution Part 2 Continued

$$R(1,1,1) = R(1,1,0) \cup R(1,1,0)R(1,1,0)^* R(1,1,0) =$$
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$$R(2,2,1) = R(2,2,0) \cup R(2,1,0)R(1,1,0)^* R(1,2,0) =$$
$$\{\{e\},b\} \cup a\{\{e\},b\}^* a = \{e,b\} \cup ab^* a$$

Hw 03 Problem 5 Solution Part 2 Continued

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Hw 03 Problem 5 Solution Part 2 Continued

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All we need is $R(1,1,2)$. So our final regex is

$$b^* \cup b^* a(\{\{e\},b\} \cup ab^* a)^* ab^*$$