

**Homework 6 DUE March 31 at 11:00 AM (No Dead Cat Extension)
THIS HOMEWORK IS TWO PAGES LONG!!!!!!!!!!!!!!!!!!!!**

1. (0 points, but if you actually miss the midterm without telling Dr. Gasarch ahead of time, you will lose 100 points on this homework) When will the midterm be (give date and time)? When will the final be (give date and time)? By when do you have to tell Dr. Gasarch that you cannot make the midterm?
2. (40 points, 10 points each part) For each of the following say if it is
 - REGULAR (in this case give a DFA or NFA or REGEX for it)
 - CONTEXT FREE BUT NOT REGULAR (in this case give a CFG for it and prove it's not regular)
 - NOT CONTEXT FREE (in this case just say NOT a CFL, no proof required).

Here are the languages:

- (a) $\{a^{2n}b^{3n} : n \in \mathbf{N}\}$.
 - (b) $\{a^{n^2+2n} : n \in \mathbf{N}\}$.
 - (c) $\{a^{n+m} : n, m \in \mathbf{N} \wedge n \geq 3m\}$
 - (d) $\{a^n b^n : n \in \mathbf{N} \wedge n \leq 100\}$.
3. (30 points, 10 points per part) Give a CFG in Chomsky Normal Form for the languages. Try to make the the number of productions as small as possible.
 - (a) $L = \{a^{16}\}$
 - (b) $L = \{a^{17}\}$
 - (c) $L = \{a^{28}\}$

GO TO NEXT PAGE FOR REST OF HOMEWORK

4. (30 points) For any $F \subseteq \mathbf{N}$, let $BILL(F)$ be the following set of strings.

$$BILL(F) = \{a^n b^n : n \in F\}$$

- (a) (8 points) TRUE or FALSE. If F is finite, then $BILL(F)$ is regular. If TRUE then show how to, given a finite F , construct a DFA for $BILL(F)$. **Give a reasonable upper bound on the number of states in the DFA as a function of F** e.g. the number of states is bounded by the largest Fibonacci prime in F (this is not the answer). If FALSE, then give a finite set F for which $BILL(F)$ is not regular and prove that it is not regular.
- (b) (7 points) TRUE or FALSE. If F is finite, then $BILL(F)$ is context free. If TRUE then show how to, given a finite F , construct a CFG for $BILL(F)$. **Give a reasonable upper bound on the number of production rules in the CFG as a function of F** e.g. the number of production rules is bounded by the largest Ramanujan prime in F (this is not the answer). (The CFG NEED NOT be in Chomsky Normal Form.) If FALSE, then give a finite set F for which $BILL(F)$ is not context free and prove that it is not context free.
- (c) (8 points) TRUE or FALSE. If F is infinite, then $BILL(F)$ is regular. If TRUE then show how to, given a infinite F , construct a DFA for $BILL(F)$. **Give a reasonable upper bound on the number of states in the DFA as a function of F** e.g. the number of states is the smallest Ramanujan prime in F (this is not the answer). If FALSE, then give a infinite set F for which $BILL(F)$ is not regular and prove that it is not regular.
- (d) (7 points) TRUE or FALSE. If F is infinite, then $BILL(F)$ is context free. If TRUE then show how to, given a infinite F , how to construct a CFG for $BILL(F)$. **Give a reasonable upper bound on the number of production rules in the CFG as a function of F** e.g. the number of states is the smallest Fermat prime in F (this is not the answer). (The CFG NEED NOT be in Chomsky Normal Form.) If FALSE, then give a infinite set F for which $BILL(F)$ is not context free. NO need to prove it.