Homework 7 DUE April 21 at 11:00 AM (Dead Cat April 23 at 11:00 AM)

1. (25 points) Find a function $T(n)$ such that

$$NP \subseteq DTIME(T(n)).$$

2. (25 points) Construct a decidable set A that is NOT in NP. You have to construct A. You cannot just invoke some theorem to show A exists. (Hint: Use Problem 1.)

3. (25 points) Let A, B, C be sets. Show that

   if $A \leq B$ and $B \leq C$ then $A \leq C$.

   (Recall that

   $X \leq Y$ means that there is a polynomial time function f such that

$$x \in X \text{ iff } f(x) \in Y.$$

)

4. (25 points) Let

$$COL_c = \{G : G \text{ is } c\text{-colorable}\}.$$

   (a) (10 points) Show $COL_3 \leq COL_4$. (That is, there exists a function f that takes a graph G and produces a graph G' such that $G \in COL_3$ iff $G' \in COL_4$.)

   (b) (15 points) Show that $COL_3 \leq COL_5$.

   (c) (0 points) Think about: is $COL_4 \leq COL_3$?