Homework 8 Morally DUE April 28 at 11:00 AM

1. (35 points) Let

$$IS_\alpha = \{G : G \text{ has an Independent Set of size } \geq \alpha n\}$$

   where n is the number of vertices in G.

   (a) (15 points) Show that $IS_{1/3}$ is NP-complete. (Hint: look at the proof that IS is NP-complete.)

   (b) (20 points) Show that

$$IS_{1/2} \leq IS_{7/8}$$

   You can assume that the graph G you are originally given has n vertices where n is divisible by 8.

   (You must give the reduction; you can't just say they are both NP-complete, though they are.)

2. (30 points) In class we did the proof that $3SAT \leq IS$.

   Let ϕ be

$$(x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_4) \wedge (x_1 \vee \neg x_3 \vee \neg x_4) \wedge (\neg x_1 \vee x_2 \vee x_3)$$

   Apply the reduction to obtain a graph G and a number k such that

   ϕ is satisfable IFF G has an ind set of size k.

   Give the graph BOTH as a drawing and FORMALLY in terms of listing its vertices and edges.

3. (35 points) A Sam-TM is one that allows the instruction

$$\delta(q, a) = (p, b, L)$$

   which means that, if the machine is in state q and is looking at a, then the state changes to p, The a is overwritten with a b, AND the head then moves left.

   Write the part of the formula that models this transition in the proof of the Cook-Levin Theorem.