Homework 10 DUE May 12 at 11:00 AM REALLY DUE
THIS HW IS THREE PAGES LONG!!!!!!!!!!!!!!!!!!!!
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1. (30 points) Write an algorithm that will, given a DFA, determines if there is some string that is accepted by it. Do the algorithm from scratch. For example, do not say something like *view the DFA as a graph and use Kruskal's Algorithm to find a spanning tree.*

   SOLUTION

   (a) Input M a DFA. Let n be the number of states.

   (b) Let $A_0 = \{s\}$. (Our plan is that A_i will be the set of states that can be reached with a $\le i$-long word.)

   (c) For $i = 1$ to n: $A_i = A_{i-1} \cup \{q : (\exists (p, \sigma) \in A_{i-1} \times \Sigma) : \delta(p, \sigma) = q\}$

   (d) If $F \cap A_n \neq \emptyset$ output Y, else output N.

   The key is that if M accepts any string then it accepts a string of length $\le n$.

   END OF SOLUTION

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2. (30 points—5 points each) In this problem we use WS1S notation. (Make sure to label states A for accept, R for Reject, S for Stupid.)

   (a) Draw a DFA for
   $$\{(x, X) : x + 1 \in X\}.$$

   (b) Draw a DFA for
   $$\{(x, X) : x + 1 \notin X\}.$$

   (c) Draw a DFA for
   $$\{(x, X) : x + 100 \in X\}.$$
   (You may use DOT DOT DOT notation.)

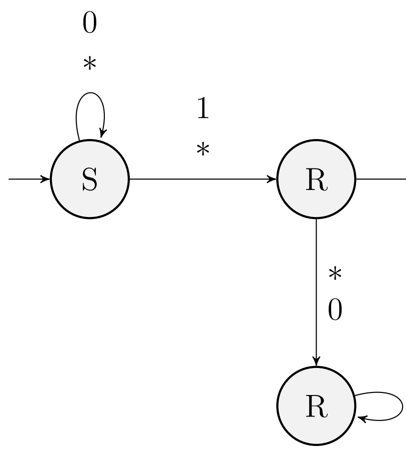
   (d) Draw a DFA for
   $$\{X : X \text{ has exactly 3 elements}\}.$$

   (e) Draw a DFA for
   $$\{X : X \text{ does NOT have exactly 3 elements}\}.$$

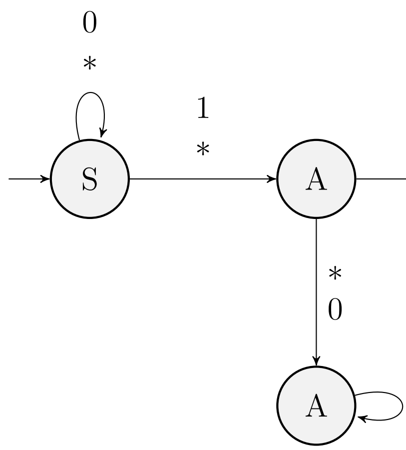
   (f) Draw a DFA for
   $$\{X : X \text{ has exactly 100 elements}\}.$$
   (You may use DOT DOT DOT notation.)

SOLUTION

(a) This is the DFA.

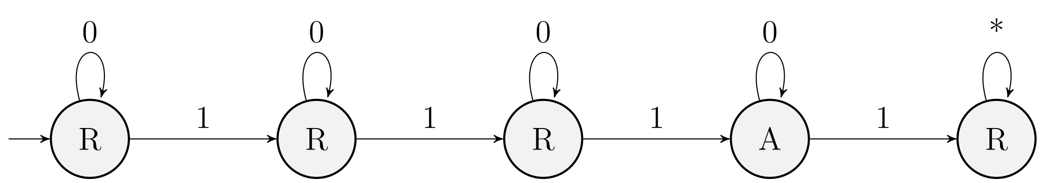


(b) This is the DFA.

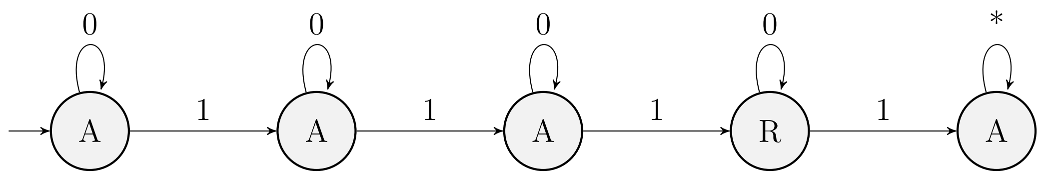


(c) OMITTED.

(d) This is the DFA.



(e) This is the DFA.



(f) OMITTED.

END OF SOLUTION

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3. (40 points) Let $M_1, M_2, \ldots,$ be a list of Turing Machines.

   (a) (20 points) Let TOT be the set of all Turing Machines that halt on ALL inputs.

   Find a computable set B of ordered triples such that:

$$TOT = \{e : (\forall x)(\exists y)[(e, x, y) \in B]\}$$

   (This means that TOT is in Π_2.)

   (b) (20 points)

   Let $HALT100$ be the set of all Turing Machines that halt on at least 100 inputs.

   Find a computable set B of ordered pairs such that:

$$HALT100 = \{e : (\exists x)[(e, x) \in B]\}$$

   (HINT: x itself will code many numbers. You need to say how you do that coding.)

SOLUTION

(a) Let TOT be the set of all Turing Machines that halt on ALL inputs.

Find a computable set B of ordered triples such that:

$$TOT = \{e : (\forall x)(\exists y)[(e, x, y) \in B]\}$$

(This means that TOT is in Π_2.)

ANSWER

$$TOT = \{e : (\forall x)(\exists y)[M_e(y) \text{ halts within } y \text{ steps}]\}$$

So

$$B = \{(e, x, y) : M_e(x) \text{ halts within } y \text{ steps}\}$$

(b) (15 points)

Let $HALT100$ be the set of all Turing Machines that halt on at least 100 inputs.

Find a computable set B of ordered pairs such that:

$$HALT100 = \{e : (\exists x)[(e, x) \in B]\}$$

(HINT: x itself will code many numbers. You need to say how you do that coding.)

ANSWER

Let $p_1, \ldots, p_{101}$ be the first 101 primes. We code 101 numbers $(a_1, \ldots, a_{101})$ by $\prod_{i=1}^{101} p_i^{a_i}$

To put this another way, the function $\pi_i(x)$ is the exp of p_i in x.

$HALT100 =$

$$\{e : (\exists x)[\text{for all } 1 \le i \le 100 \ M_e(\pi_i(x)) \text{ halts in} \le \pi_{101}(x) \text{ steps }]$$

END OF SOLUTION