CMSC452 Final

1. This is an open-everything exam. You can use anything except ask another person. Caution: if you copy from the web or elsewhere mindlessly you will probably get it wrong.

2. There are 3 problems which add up to 60 points. There was a take-home part that was worth 40 points.

3. All problems have the next page or two blank. You may use the page a problem is on and the blank pages right after it for your answer.

4. The exam is Thursday May 13 8:00PM-10:15PM unless you have contacted me to make other arrangements. So the exam is 2 hours and 15 minutes.

5. For each question show all of your work and use LaTeX or write VERY NEATLY. Clearly indicate your answers. No credit for illegible answers.

6. Please write out the following statement: I pledge on my honor that I will not give or receive any unauthorized assistance on this examination.
1. (20 points) Give an example of each of the following.

   **NO PROOF REQUIRED**

   (a) (7 points) A decidable set that is thought to NOT be in NP.

   (b) (7 points) A set that is in NP, but whether it is in P or NP-complete is unknown.

   (c) (6 points) ANOTHER set that is in NP, but whether it is in P or NP-complete is unknown.
2. (20 points) In this problem we modify our Turing Machines to they can (1) take elements of $\mathbb{Z}$ as input and (2) have elements of $\mathbb{Z}$ as output. (They can still output Y and N when used to decide sets.)

Let $g$ be a COMPUTABLE function from $\mathbb{Z}$ to $\mathbb{Z}$ such that

$$
\cdots \leq g(-2) \leq g(-1) \leq g(0) \leq g(1) \leq g(2) \leq \cdots
$$

Let

$$
Y = \{ \ldots, g(-2), g(-1), g(0), g(1), g(2), \ldots \}
$$

Is $Y$ decidable?

If YES then give an algorithm that decides $Y$.

If NO then prove that it is not.

Do this problem on this page and the next page.
3. (20 points)

For this problem you should assume $n$ is a large power of 2, and $\Sigma = \{a, b\}$.

(a) (5 points) Let $w$ be a string of length $n$. Show that there is a Chomsky Normal Form CFG for $\{w\}$ with $O(n)$ rules.

(b) (5 points) Let $w = a^n$. Show that there is a Chomsky Normal Form CFG for $\{w\}$ with $O(\log n)$ rules.

(c) (10 points) Show that there exists a string $w$ such that ANY Chomsky Normal Form CFG for $\{w\}$ requires MUCH MORE than $\log n$ rules.