1. (0 points, but if you actually miss the midterm without telling Dr. Gasarch ahead of time, you will lose 100 points on this homework) When will the midterm be (give date and time)? When will the final be (give date and time)? By when do you have to tell Dr. Gasarch that you cannot make the midterm?

2. (20 points) The alphabet is \{a\}.

   (a) (10 points) Give a small CFG in Chomsky Normal Form for

   \( \{a^{13}\} \)

   Hint: Use 12 = \(2^3 + 2^2 + 2^0\).

   (b) (10 points) Describe how to obtain, for any \(n\) (NOT just a power of 2), a small CFG for

   \( \{a^n\} \)

   Hint: First step is to write \(n\) as a set of powers of 2.

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3. (20 points) For this problem \( \Sigma = \{a, b, c\} \).

Give a CFG for

\[ L = \{a^n b^n c^n : n \in \mathbb{N}\} \]
4. (20 points) Let $\Sigma = \{a, b\}$. Let $n \in \mathbb{N}$. Let

$$L_n = \{w : |w| \leq n\}.$$

(a) (6 points) Give a regex for $L_n$. (You may use DOT DOT DOT.) Try to make it as short as possible. What is its length? You may use $O$-notation. THINK ABOUT (but do not hand in) is your result the best possible? Can you make a shorter regex?

(b) (7 points) Give a trex for $L_n$. (You may use DOT DOT DOT.) Try to make it as short as possible. What is its length? You may use $O$-notation. THINK ABOUT (but do not hand in) is your result the best possible? Can you make a shorter trex?

(c) (7 points) Give a CFG in Chomsky Normal Form for $L_n$. Try to make it as short as possible. What is its length? (It should be $\ll n$.) THINK ABOUT (but do not hand in) is your result the best possible? Can you make a smaller Chomsky Normal Form CFG?

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5. (20 points) For a given alphabet $\Sigma$, if $w \in \Sigma^*$ then $w^R$ is $w$ written in reverse. For example

$$aaab^R = baaa.$$ 

If $L \subseteq \Sigma^*$ then

$$L^R = \{w^R : w \in L\}.$$ 

(a) (10 points) Show that if $L$ is a CFL then $L^R$ is a CFL 
(b) (10 points) For this problem we assume all CFGs are in Chomsky Normal Form. Is there a language $L$ such that both of the following hold:

- There is a CFG for $L$ of size $O(n)$.
- Every CFG for $L^R$ requires $\Omega(n^2)$.

If SO then present such an $L$ (no proof required).
If NOT then prove that no such $L$ exists.

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6. (20 points) (I am reminding you what SUBSEQ is even though it was on a prior hw.)

For an alphabet \( \Sigma \) let \( x \in \Sigma^* \). Then

\( \text{SUBSEQ}(x) \) is the set of all strings formed by removing some letters (possibly 0, possibly all of them)

**Example**

\[
\text{SUBSEQ}(abbaa) = \\
\{e, a, b, aa, ab, ba, bb, aaa, abaa, abba, baa, bba\} \cup \\
\{abaa, abba, bbaa, abbaa\}.
\]

Let \( L \) be a language.
Then

\[
\text{SUBSEQ}(L) = \bigcup_{x \in L} \text{SUBSEQ}(x).
\]

And now FINALLY for the question.

Show that if \( L \) is a CFL then \( \text{SUBSEQ}(L) \) is a CFL.