1. (0 points) When is the final? (HINT: It's Thursday May 13, 8:00PM-10:15PM AND NOTICE THAT IT'S PM.)

2. (30 points) READ the slides on finding a certificate that a number is prime (They are part of the lecture on Factoring prob not NPC).

As part of the certificate that a number is prime, you need certificates that smaller numbers are prime.

In the certificate that 100103 is prime, list all of the smaller primes that you will need certificates for. (If you get to 2, you do not need to recurse below that.)

3. (a) (30 points) $L \in DTIME(2^{O(n)})$ if there is a constant $a$ and an algorithm (psuedocode is fine) that (1) decides $L$, and (2) on inputs of length $n$ runs in time $2^{an}$ time.

Show that if $L \in DTIME(2^{O(n)})$ then $L^* \in DTIME(2^{O(n)})$.

(b) (0 points, but think about) Is there some function $T(n)$ such that $DTIME(T(n))$ is NOT closed under *? (The $T(n)$ can have $O$-of in it.)

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4. (40 points) **Definition** Let $f$ be a function.

$f$ is **strictly increasing** if, for all $x < y$, $f(x) < f(y)$.

$f$ is **monotone increasing** if, for all $x < y$, $f(x) \leq f(y)$.

If $f$ is any function then

\[ \text{image}(f) = \{ y : (\exists x) [f(x) = y] \}. \]

And now FINALLY the problem

(a) (20 points) Let $f$ be a computable function with domain $\mathbb{N}$ and codomain $\mathbb{N}$ that is strictly increasing. Prove that image($f$) is decidable.

(b) (20 points) Let $f$ be a computable function with domain $\mathbb{N}$ and codomain $\mathbb{N}$ that is monotone increasing. Prove that image($f$) is decidable.