CMSC452 Midterm

1. This is an open-everything exam. You can use anything except ask another person. **Caution:** if you copy from the web or elsewhere mindlessly you will probably get it wrong.

2. There are 5 problems which add up to 100 points.

3. The exam is March 23 from 8:00PM until 10:15PM unless you have contacted me to make other arrangements. So the exam is 2 hours and 15 minutes

4. For each question show all of your work and **use LaTeX or write VERY NEATLY. Clearly indicate** your answers. No credit for illegible answers.

5. Please write out the following statement: *I pledge on my honor that I will not give or receive any unauthorized assistance on this examination.*
1. (20 points) Give an example of each of the following. NO PROOF REQUIRED

(a) (10 points) A Context Sensitive Language that is not a Context Free Language.
Give the language AND the Context Sensitive Grammar for it.
(b) (10 points) A regular language $L$ over the alphabet $\Sigma = \{a\}$ such that any DFA for $L$ requires $\geq 100$ states.
2. (20 points)

The alphabet is \( \{a\} \). Let

\[
L = \{a^i : i \neq 200\}
\]

Does there exist an NFA for \( L \) with less than 100 states? If so then draw the NFA; you may use DOT DOT DOT (You DO NOT have to prove that the NFA works.) If not then PROVE there is no such NFA.

You may answer this problem on this page and the next page.
3. (20 points) Prove that the following language is NOT REG-ULAR.

\[ L = \{ww : w \in \{a, b\}^*\}. \]

You may answer this problem on this page and the next page.
4. (20 points) Let the alphabet be \( \{a, b, c, d\} \). Give a Context Free Grammar for

\[
\{a^m wd^m : m < n \text{ and } w \in \{b, c\}^*\}
\]

No proof required and it DOES NOT have to be in Chomsky Normal Form.

You may answer this problem on this page and the next page.
5. (20 points)

Let $\Sigma = \{a, b\}$. For every $n \in \mathbb{N}$ let

$L_n = \{ww : |w| = n\}$.

For example

$L_2 = \{aaaa, abab, baba, bbbb\}$

Show that any DFA for $L_n$ requires $\geq 2^n$ states.

(NOTE - DO NOT give me a DFA that uses that many states. That is IRRELEVANT to this problem. You need to show that ANY DFA REQUIRES $\geq 2^n$ states.)

You may answer this problem on this page and the next page.
Scratch Paper