BILL, RECORD LECTURE!!!!

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Deterministic Finite Automata (DFA): Closure Properties

Regular Lang Closed Under Complimentation

How do you compliment a regular language?

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Example How do you compliment a^*?

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Compliment An expression of admiration.

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Compliment An expression of admiration.
Complement The complement of L is $\Sigma^* - L$.

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Note If DFA for L has n states then DFA for $\overline{L}$ has n states.

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$$(Q_1 \times Q_2, \Sigma, \delta, (s_1, s_2), F)$$

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Note The number of states in DFA for $L_1 \cup L_2$ is $n_1 n_2$.

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Summary of Closure Properties and Proofs

X means **Can't Prove Easily**
$n_1 + n_2$ (and similar) is number of states in new machine if L_i reg via n_i-state machine.

Closure Property	DFA
$L_1 \cup L_2$	$n_1 n_2$
$L_1 \cap L_2$	$n_1 n_2$
$L_1 \cdot L_2$	X
$\overline{L}$	n
L^*	X

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