BILL, RECORD LECTURE!!!!

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Deterministic Finite Automata (DFA): Closure Properties
How do you compliment a regular language?

Example

I find the way all of your strings have only $a$'s so lovely!

Compliment

An expression of admiration.

Complement

The complement of $L$ is $\Sigma^* - L$. 
How do you compliment a regular language?

Example How do you compliment $a^*$?
How do you compliment a regular language?

**Example** How do you compliment \( a^* \)?

I find the way all of your strings have only \( a \)'s so lovely!
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*I find the way all of your strings have only $a$’s so lovely!*

**Compliment** An expression of admiration.

**Complement** The complement of $L$ is $\Sigma^* - L$. 
How do you complement a regular language?

Informally

Swap the final and non-final states.

Formally

If $L$ is regular via $(Q, \Sigma, \delta, s, F)$ then $L$ is regular via $(Q, \Sigma, \delta, s, Q - F)$.

Note

If DFA for $L$ has $n$ states then DFA for $L$ has $n$ states.
How do you complement a regular language?

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then $\overline{L}$ is regular via

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Regular Lang Closed Under Complementation

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Regular Lang Closed Under Union

IF \( L_1, L_2 \) are regular we want to show that \( L_1 \cup L_2 \) is regular.
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IF $L_1, L_2$ are regular we want to show that $L_1 \cup L_2$ is regular.

**Informally** Create a DFA that runs both the DFA for $L_1$ and $L_2$ at the same time.
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\[
(Q_1 \times Q_2, \Sigma, \delta, (s_1, s_2), F_1 \times Q_2 \cup Q_1 \times F_2)
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where

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and

Note: The number of states in DFA for $L_1 \cup L_2$ is $n_1n_2$. 
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Note The number of states in DFA for $L_1 \cap L_2$ is $n_1 n_2$. 
Regular Lang Closed Under Concatenation?

**Question** Is the following true?

IF $L_1, L_2$ are regular then $L_1 \cdot L_2$ is regular.
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Regular Lang Closed Under $\ast$?

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Question Is the following true? IF $L$ is regular then $L^*$ is regular.

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Regular Lang Closed Under $*$?

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**YES**

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**Regular Lang Closed Under ∗?**

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X means **Can’t Prove Easily**

$n_1 + n_2$ (and similar) is number of states in new machine if $L_i$ reg via $n_i$-state machine.

<table>
<thead>
<tr>
<th>Closure Property</th>
<th>DFA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1 \cup L_2$</td>
<td>$n_1n_2$</td>
</tr>
<tr>
<td>$L_1 \cap L_2$</td>
<td>$n_1n_2$</td>
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<td>$n_1n_2$</td>
</tr>
<tr>
<td>$\overline{L}$</td>
<td>X</td>
</tr>
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<td>$L^*$</td>
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</tbody>
</table>
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