# BILL, RECORD LECTURE!!!!

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# Tricks for Divisibility and DFA's

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We feed a number into a DFA right-to-left: $d_0$, then $d_1$ etc.

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**Pf**

$$d_n \times 10^n + \cdots + d_1 \times 10 + d_0 = 10(d_n \times 10^{n-1} + \cdots + d_1) + d_0 \equiv d_0.$$

# DFA for Mod 2

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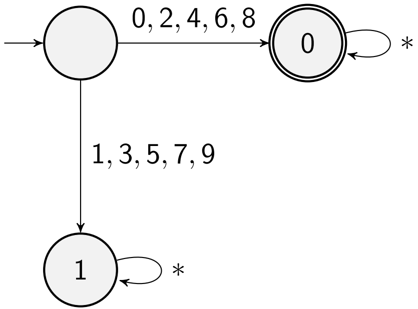

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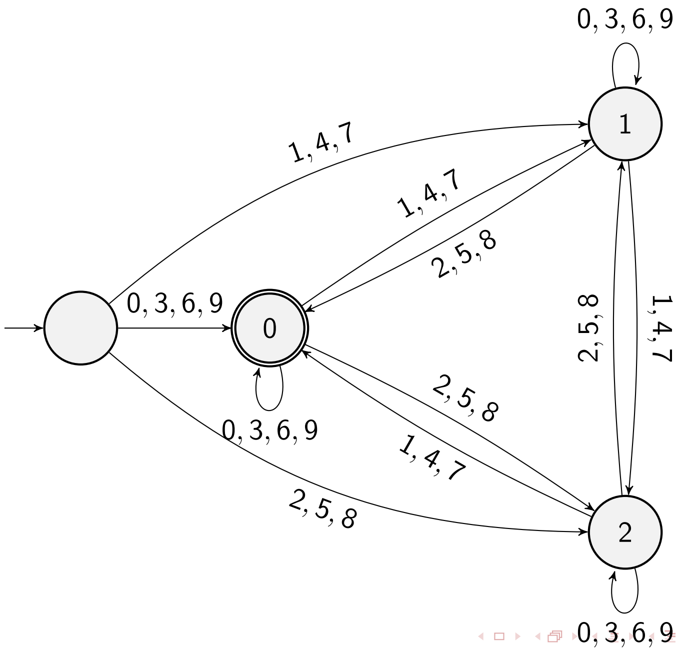

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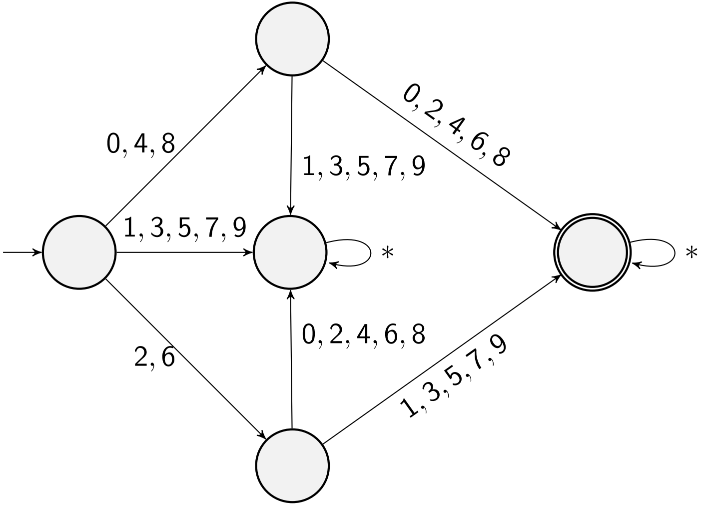

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Mod 3: Pattern is 1,1,1,1,..., DFA tracked sum of digits mod 3.
Mod 4: Pattern is 1,2,0,0,0,..., DFA only cared about first 2 digits.

# Tricks for Mod 5 and Mod 6

These may be on a HW.

# Trick for Mod 11. All $\equiv$ are Mod 11

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Proof may be on HW or Midterm or Final or some combination.

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**Classifier** If end in $(i, 0)$ or $(i, 1)$ then number is $\equiv i$.

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Pattern is $1, 3, 2, 6, 4, 5, 1, 3, 2, 6, 4, 5, 1, \ldots$.

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Pattern is $1, 3, 2, 6, 4, 5, 1, 3, 2, 6, 4, 5, 1, \ldots$.
Can we use this?

# Using the Divide by 7 Trick

Want to know what 3876554 is mod 7. All arith is mod 7.
$4 \times 1 + 5 \times 3 + 5 \times 2 + 6 \times 6 + 7 \times 4 + 8 \times 5 + 3 \times 1$

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$\equiv 4+1+3+1+0+5+3 \equiv (4+3+1)+(3+1+5+3) \equiv 1+5 \equiv 6.$

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So there are $7 \times 6 = 42$ states.

# Is the Method a Trick?

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**NO** A human can't do it easily- the pattern is not like 1,1,1,... or mostly 0's.

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Might make it a HW to do as a table.

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