BILL AND NATHAN RECORD LECTURE!!!!

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HW02 SOLUTION

HW02 Problem 2. n is even. Give a DFA for

$$\left\{ w : |w| = n \text{ and } \#_a(w) = \#_b(w) = \frac{n}{2} \right\}$$

How many states as a function of n?

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How many states as a function of n?**SOLUTION**

states keep track of how length and how many a'a and b's.

$Q = \{0, \ldots, n\} \times \{0, \ldots, \frac{n}{2}\} \times \{0, \ldots, \frac{n}{2}\} \cup DUMP$. $s = (0, 0, 0)$.

$$\delta((i,j,k),a) = \begin{cases} (i+1, j+1, k) & \text{if } i \le n-1 \text{ and } j \le \frac{n}{2} - 1 \\ DUMP & \text{if } i = n \text{ or } j = \frac{n}{2} \end{cases}$$

$$\delta((i,j,k),b) = \begin{cases} (i+1, j, k+1) & \text{if } i \le n-1 \text{ and } k \le \frac{n}{2} - 1 \\ DUMP & \text{if } i = n \text{ or } k = \frac{n}{2} \end{cases}$$

$\delta(DUMP, \sigma) = DUMP$

$F = \{(n, \frac{n}{2}, \frac{n}{2})\}$

Number of States $= (n+1)(\frac{n}{2} + 1)^2 + 1$

HW02 Problem 3

Draw an NFA for the language $a^* b^* \cup bba^* b$.

SOLUTION

Omitted.

END OF SOLUTION

HW02 Problem 4

$\Sigma = \{0, 1, 2, 3, 4, 5, 6\}$. We view strings as numbers in base 7. We read them least-sig-digit-first. Write the DFA-classifier that tells us what a number is mod 5.
How many STATES your classifier has.

SOLUTION

We need to look at powers of 7 mod 5.
$7^0 \equiv 1$
$7^1 \equiv 2$
$7^2 \equiv 4$
$7^3 \equiv 3$
$7^4 \equiv 1$
So will need to keep track of position mod 4.

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Next slide has the DFA.

HW02 Problem 4

Our DFA has
$Q = \{0, \ldots, 6\} \times \{0, 1, 2, 3\}$.
$s = (0, 0)$

$$\delta((i,j),\sigma) = \begin{cases} (i+\sigma \pmod 5, j+1 \pmod 4) & \text{if } j \equiv 0 \pmod 4 \\ (i+2\sigma \pmod 5, j+1 \pmod 4) & \text{if } j \equiv 1 \pmod 4 \\ (i+4\sigma \pmod 5, j+1 \pmod 4) & \text{if } j \equiv 2 \pmod 4 \\ (i+3\sigma \pmod 5, j+1 \pmod 4) & \text{if } j \equiv 3 \pmod 4 \end{cases}$$

If $w \equiv 0 \pmod 5$ then ends in one of $\{(0, j) : j \in \{0, 1, 2, 3\}\}$.
If $w \equiv 1 \pmod 5$ then ends in one of $\{(1, j) : j \in \{0, 1, 2, 3\}\}$.
If $w \equiv 2 \pmod 5$ then ends in one of $\{(2, j) : j \in \{0, 1, 2, 3\}\}$.
If $w \equiv 3 \pmod 5$ then ends in one of $\{(3, j) : j \in \{0, 1, 2, 3\}\}$.
If $w \equiv 4 \pmod 5$ then ends in one of $\{(4, j) : j \in \{0, 1, 2, 3\}\}$.
END OF SOLUTION

HW02 Problem 5

L is **Saadiq-regular** if there is an NFA M such that

$$L = \{x : \text{the set of final states } M(x) \text{ COULD reach is } \geq 2\}.$$

Show that if L is Saadiq-regular then L is regular.

HW02 Problem 5

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$$L = \{x : \text{the set of final states } M(x) \text{ COULD reach is } \geq 2\}.$$

Show that if L is Saadiq-regular then L is regular.

SOLUTION

Let L be Saadiq-regular via NFA M.

Do the same construction we use to convert an NFA to a DFA M'.

We just CHANGE the set of final states for M'.

Recall that for the states of M' is the powerset of the states in M.

For Saadiq-reg we will take F' to be

All subsets of Q that have ≥ 2 elements of F.

END OF SOLUTION

HW02 Problem 6

Let $M = (Q, \Sigma, \Delta, s, F)$ be an NFA for L
Construct an NFA for L^* from the NFA for L.
SOLUTION
$M = (Q \cup \{s'\}, \Sigma, \Delta', s', F \cup \{s'\}))$ where
If $q \in Q$ and $\sigma \in \Sigma$ then $\Delta'(q, \sigma) = \Delta(q, \sigma)$.
$\Delta(s', e) = s$.
If $q \in F$ then $\Delta'(q, e) = s'$.
END OF SOLUTION