BILL AND NATHAN START RECORDING

HW04 Solutions

Problem 2a

$$L = \{w : \#_a(w) \equiv 0 \pmod n \vee \#_b(w) \equiv 0 \pmod n \vee \#_c(w) \equiv 0 \pmod n\}$$

1) Give a DFA for L with n^3 states. Give it as a table.

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$Q = [n] \times [n] \times [n]$. $s = (0,0,0)$. $F = \{(a,b,c) : abc = 0\}$.
The intuition is that we keep track of number of a's mod n AND number of b's mod n AND number of c's mod n.

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Table on next page

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This NFA has $3n + 1$ states.

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A similar argument can be made for $j = j'$ and $k = k'$.

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It is easy to draw an $O(n)$ state DFA for L.
$L^R = \Sigma^* a \Sigma^n$ which we have shown before requires 2^{n+1} states.

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NOT REGULAR proof omitted but it is is similar to proving

$$\{a^{n^2} : n \in \mathrm{N}\}$$

is not regular which we did in class.

Problem 5b

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We omit the rest.

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This is just a^*.

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