BILL AND NATHAN START RECORDING

HW05 Solutions

HW05, Problem 2

Give a small CFG in Chomsky Normal Form for

$$\{a^{13}\}$$

Hint: Use $12 = 2^3 + 2^2 + 2^0$.

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Sketch Write a CFG for a^8, a^4, and a^0 and then do concat.

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Hint: Use $12 = 2^3 + 2^2 + 2^0$.

SOLUTION

Sketch Write a CFG for a^8, a^4, and a^0 and then do concat.

Describe how to obtain, for any n (NOT just a power of 2), a small CFG for

$$\{a^n\}$$

Sketch Let $n = 2^{n_1} + \cdots + 2^{n_k}$. Write a CFG for each $\{a^{2^{n_i}}\}$ and then do th concat.

HW05, Problem 3

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SOLUTION

L is a union of the following

(1) $\{a^m b^n c* : m < n\}$. We did $\{a^m b^n : m < n\}$ in class- so concat with c^*.

(2) $\{a^m b^n c* : m > n\}$. Similar to (1).

(3) $\{a^* b^n c^p : n < p\}$. Similar to (1).

(4) $\{a^* b^n c^p : n > p\}$. Similar to (1).

(5) $\{a^m b^* c^p : m < p\}$. We do this on the next slide

(6) $\{a^m b^* c^p : m > p\}$. Similar to (5).

(7)

$\{a, b, c\}^* ba\{a, b, c\}^* \cup \{a, b, c\}^* ca\{a, b, c\}^* \cup \{a, b, c\}^* cb\{a, b, c\}^*$

You would need to do the CFG's for all of these and then put them all together.

HW05, Problem 3 (cont)

$\{a^m b^* c^p : m < p\}$.

HW05, Problem 3 (cont)

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$S \to ATCC$

$T \to ATC \quad | \, e$

$T \to TC \quad | \, e$

$T \to B$

$B \to Bb \quad | \, e$

$A \to a$

$C \to c$

HW05, Problem 4

$$L_n = \{w : |w| \leq n\}.$$

1) Give a regex for L_n. (You may use DOT DOT DOT.) Try to make it as short as possible. What is its length? You may use O-notation. THINK ABOUT (but do not hand in) is your result the best possible? Can you make a shorter regex?

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$\{a, b, e\}\{a, b, e\} \cdots \{a, b, e\}$ where there are n of the $\{a, b, e\}$.

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$\{a, b, e\}\{a, b, e\} \cdots \{a, b, e\}$ where there are n of the $\{a, b, e\}$. This is $O(n)$ long.

There is no shorter regex. The regex cannot have a * in it since that would generate an infinite number of strings. Hence if the regex is of length ℓ then it can only generate strings of length $\leq n$.

HW05, Problem 4 (cont)

$$L_n = \{w : |w| \leq n\}.$$

2) Give a trex for L_n. (You may use DOT DOT DOT.) Try to make it as short as possible. What is its length? You may use O-notation. THINK ABOUT (but do not hand in) is your result the best possible? Can you make a shorter trex?

HW05, Problem 4 (cont)

$$L_n = \{w : |w| \leq n\}.$$

2) Give a trex for L_n. (You may use DOT DOT DOT.) Try to make it as short as possible. What is its length? You may use O-notation. THINK ABOUT (but do not hand in) is your result the best possible? Can you make a shorter trex?

SOLUTION

$\{a, b, e\}^n$.

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$\{a, b, e\}^n$.

This is $O(\log n)$ long.

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SOLUTION

$\{a, b, e\}^n$.

This is $O(\log n)$ long.

I am sure this is optimal, though I do not know how to prove it!

HW05, Problem 5

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THINK ABOUT (but do not hand in) is your result the best possible? Can you make a smaller Chomsky Normal Form CFG?
SOLUTION
Use the technique from problem 2 to get $\lg n + O(1)$ rules that generate A^n, Then add:
$A \to a \mid b \mid e$.

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THINK ABOUT (but do not hand in) is your result the best possible? Can you make a smaller Chomsky Normal Form CFG?

SOLUTION

Use the technique from problem 2 to get $\lg n + O(1)$ rules that generate A^n, Then add:
$A \to a \mid b \mid e$.
One make a shorter Chomsky normal form CFG, but the proof is beyond the scope of THIS version of the course. If you are curious look up the proof of the pumping lemma for CFL's and use similar techniques.

HW05, Problem 5a

Show that if L is a CFL then L^R is a CFL For this problem we assume all CFGs are in Chomsky Normal Form.

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SOLUTION

For every rule of the form

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SOLUTION

For every rule of the form

$$A \to BC$$

replace it by

$$A \to CB$$

HW05, Problem 5b

Is there a langauge L such that both of the following hold:
1) There is a CFG for L of size $O(n)$, AND
2) Every CFG for L^R requires $\Omega(n^2)$.
If SO then present such an L (no proof required).
If NOT then prove that no such L exists.

HW05, Problem 5b

Is there a langauge L such that both of the following hold:
1) There is a CFG for L of size $O(n)$, AND
2) Every CFG for L^R requires $\Omega(n^2)$.
If SO then present such an L (no proof required).
If NOT then prove that no such L exists.

SOLUTION

There is NO such L.

HW05, Problem 5b

Is there a langauge L such that both of the following hold:
1) There is a CFG for L of size $O(n)$, AND
2) Every CFG for L^R requires $\Omega(n^2)$.
If SO then present such an L (no proof required).
If NOT then prove that no such L exists.

SOLUTION

There is NO such L.

Our construction does not increase the number of rules.

HW05, Problem 6a

Show that if L is a CFL then $\mathrm{SUBSEQ}(L)$ is CFL.

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SOLUTION

Let L be a CFL with CFG in Chomsky normal form grammar G.
Here is a CFL for $\mathrm{SUBSEQ}(L)$:
Add the following rules to G:
For every rule of the form

$$A \to \sigma$$

HW05, Problem 6a

Show that if L is a CFL then $\mathrm{SUBSEQ}(L)$ is CFL.

SOLUTION

Let L be a CFL with CFG in Chomsky normal form grammar G.
Here is a CFL for $\mathrm{SUBSEQ}(L)$:
Add the following rules to G:
For every rule of the form

$$A \to \sigma$$

add the rule

$$A \to e.$$