BILL AND NATHAN RECORD LECTURE!!!!

BILL RECORD LECTURE!!!

Nondeterministic Finite Automata (NFA): Closure Properties

Terminology: Reg Langs

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We will keep track of number-of-states.

Reg Langs Closed Under Complementation

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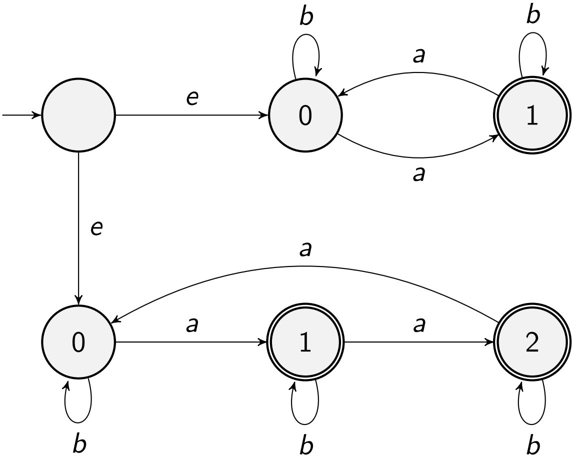
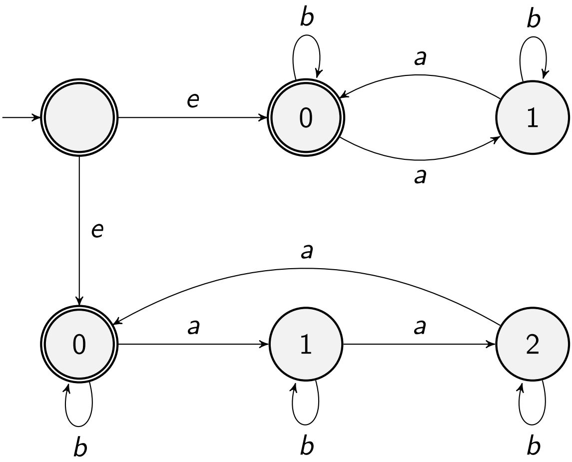
See next slide.

$\{a^n : n \not\equiv 0 \pmod 6\}$



Final and Non-final States Swapped



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No. There are langs L where:

- there is an NFA for L is size n.
- any NFA for $\overline{L}$ is of size $\geq\sim 2^n$.

Reg Langs Closed Under Union-Intuition

IF L_1, L_2 are reg we want to show that $L_1 \cup L_2$ is reg.

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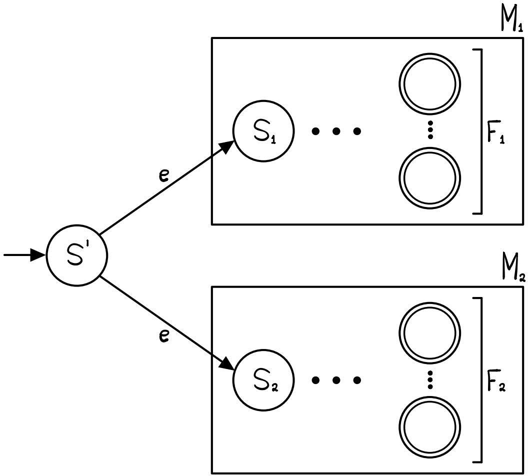
Reg Langs Closed Under Union-Picture



Reg Langs Closed Under Union-Formally

Formally If L_1 is reg via NFA

$$(Q_1, \Sigma, \Delta_1, s_1, F_1). \text{ We will take } |Q_1| = n_1 \ .$$

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$$(\{s'\} \cup Q_1 \cup Q_2, \Sigma, \Delta', s', F_1 \cup F_2).$$

where for $i = 1$ or 2,
If $q \in Q_i$, $\sigma \in \Sigma \cup \{e\}$ then $\Delta'(q, \sigma) = \Delta_i(q, \sigma)$.

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Note When we did closure using DFA's, we got $n_1 n_2$.

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Answer Option 2: Can do with NFAs but gets $n_1 n_2$ states.
It is a cross product construction. Next Slide.

Reg Langs Closed Under Intersection: Proof

Let $M_1 = (Q_1, \Sigma, \Delta_1, s_1, F_1)$ be an NFA for L_1
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From M_1 and M_2 construct an NFA M for $L_1 \cap L_2$.

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Reg Langs Closed Under Concat-Intuitively

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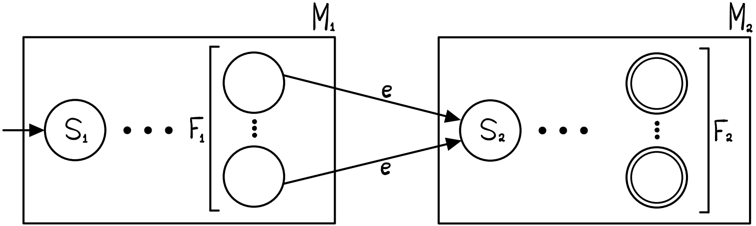

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If $q \in Q_2$, $\sigma \in \Sigma \cup \{e\}$ then $\Delta'(q, \sigma) = \Delta_2(q, \sigma)$.
Number of states: $n_1 + n_2$.

Reg Langs Closed Under $*$?-Intuition-1st Try

Have an e-transition from final states of M to start state of M.

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Next slide has a generic picture of this approach.

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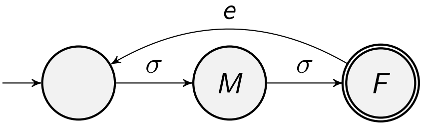
Spoiler Alert This will not work.

Reg Langs Closed Under $*$?-Picture-1st Try



What Goes Wrong with 1st Try?

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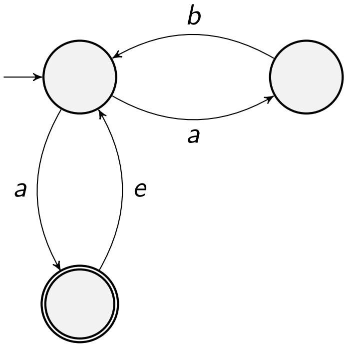
We want e to be accepted.
Next slide has an NFA where this does not work.

What Goes Wrong with 1st Try?-Picture



Reg Langs Closed Under $*$?-Intuition-2nd Try

Have an e-transition from final states of M to start state of M AND make s a final state.

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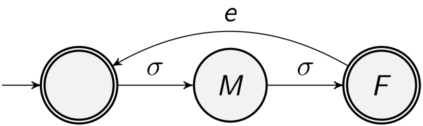

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Might accept too much.

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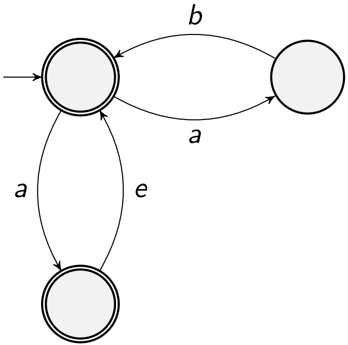

Reg Langs Closed Under $*$?-Intuition-3rd Try

Have an e-transition from final states of M to a NEW start state of M. That NEW start state is a final state and has an e-trans to old start state.

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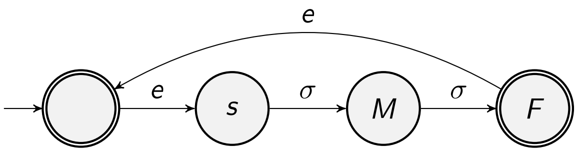

Reg Langs Closed Under $*$?-Formally

Might be a HW or exam question.

Summary of Closure Properties and Proofs

X means **can't prove easily**

$n_1 + n_2$ (and similar) is number of states in new machine if L_i reg via n_i-state machine.

Closure Property	DFA	NFA
$L_1 \cup L_2$	$n_1 n_2$	$n_1 + n_2 + 1$
$L_1 \cap L_2$	$n_1 n_2$	$n_1 n_2$
$L_1 \cdot L_2$	X	$n_1 + n_2$
$\overline{L}$	n	X
L^*	X	$n + 1$

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