BILL, RECORD LECTURE!!!!

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Nondeterministic Finite Automata (NFA)

An Interesting Example of a DFA

In breakout rooms do the following and keep track of how many states.
$\Sigma^* a$
$\Sigma^* a \Sigma$
$\Sigma^* a \Sigma^2$

$\Sigma^* a \Sigma^2$

https://www.cs.umd.edu/users/gasarch/COURSES/452/S21/notes/dfa3.JPG

$\Sigma^* a \Sigma^2$

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The number of states is 8.

$\Sigma^* a \Sigma^2$

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More generally:
$\Sigma^* a \Sigma^i$ can be done with 2^{i+1} states.

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Prove for $\Sigma^* a \Sigma^3$, with a table.

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Might be on $2^{\{\text{HW, MIDTERM, FINAL}\}}$.

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Is there a smaller DFA for $\Sigma^* a \Sigma^i$? Fewer than 2^{i+1} states?

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We may prove this later.

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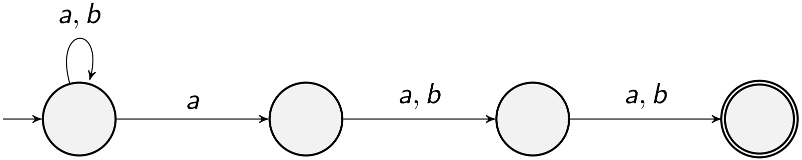
We now use NFA's informally.

NFA for $\Sigma^* a \Sigma^2$



$\{w : \#_a \equiv 0 \pmod 3) \lor w : \#_b \equiv 0 \pmod 4)\}$

The DFA for this requires 12 states. Can we do this with a smaller NFA?

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YES - next slide.

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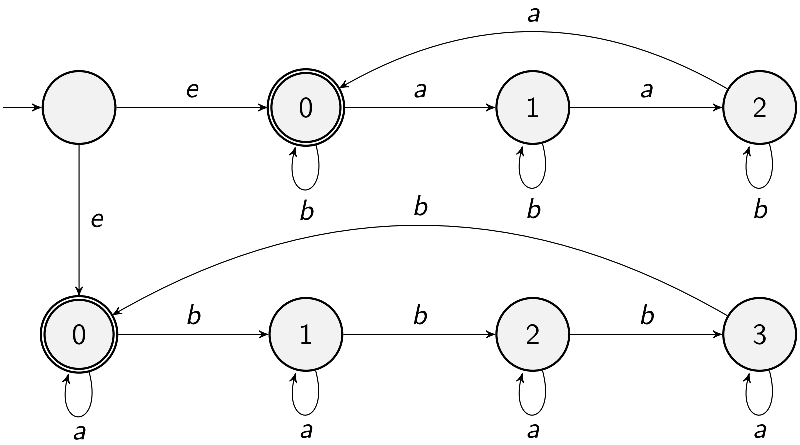

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NO. Proof similar to that for DFA. Will come back to this after we define NFA rigorously.

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Or might be on HW-MID-FINAL.

$\{a^n : n \not\equiv 0 \pmod{15}\}$

Note A DFA for this **requires** 15 states. Can a smaller NFA recognize it? VOTE.

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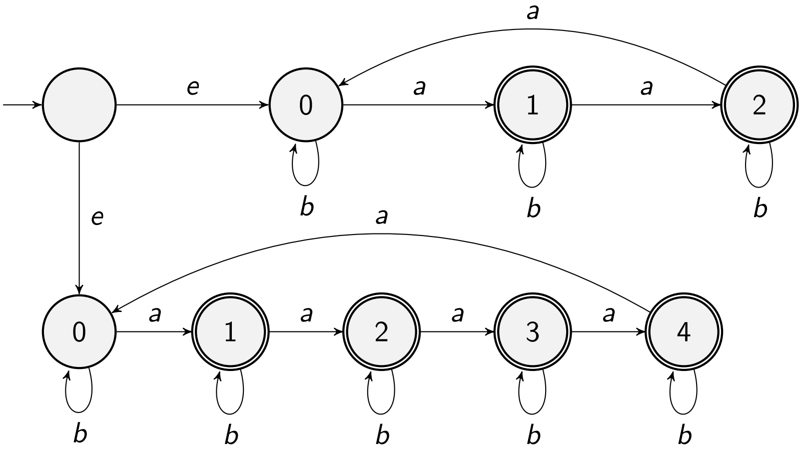

$\{a^n : n \not\equiv 0 \pmod{15}\}$

Prove that the NFA in the last slide works.
Need

$$(n \not\equiv 0 \pmod 3 \vee n \not\equiv 0 \pmod 5) \implies n \not\equiv 0 \pmod{15}$$

Take the contrapositive

$$n \equiv 0 \pmod{15} \implies (n \equiv 0 \pmod 3 \wedge n \equiv 0 \pmod 5)$$

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NFA's Intuitively

1. An NFA is a DFA that can guess.
2. NFAs do not really exist.
3. Good for $\cup$ since can guess which one.
4. An NFA accepts iff SOME guess accepts.

NFA Formally

Def An **NFA** is a tuple $(Q, \Sigma, \Delta, s, F)$ where:

1. Q is a finite set of **states**.
2. Σ is a finite **alphabet**.
3. $\Delta : Q \times (\Sigma \cup \{e\}) \to 2^Q$ is the *transition function*.
4. $s \in S$ is the **start state**.
5. $F \in Q$ is the set of **final states**.

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Def If M is an NFA and $x \in \Sigma^*$ then $\boldsymbol{M(x)}$ **accepts** if when you run M on x some sequence of guesses end up in a **final state**.

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Note When you run $M(x)$ and choose a path one of three things can happen: (1) ends in a final state, (2) ends in a non-final state, (3) cannot process.
Def If M is an NFA then $L(M) = \{x : M(x) \text{ accepts } \}$.

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Every NFA-lang a DFA-lang!

Thm If L is accepted by an NFA then L is accepted by a DFA.
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We are nowhere near done. Next slide.

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Thm If L is accepted by an NFA on n states then L is accepted by a DFA on $\le 2^n$ states.
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Key The DFA will keep track of the **set** of states that the NFA could have been in.

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If NFA accepts on some path then in the DFA you will be in a state which is a set-of-states, which includes a final state from the NFA. If the DFA accepts then there was some say for the NFA to accept.

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