BILL AND NATHAN RECORD LECTURE!!!!

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Proving a Lang is Not Regular

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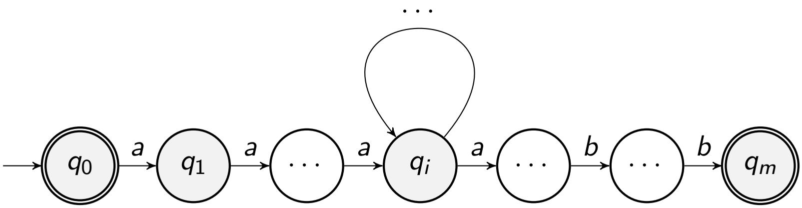
$a^j \cdot a^{m-i} b^m = a^{m+j-i} b^m \notin L_1$, so $M(a^j \cdot a^{m-i} b^m) = q \notin F$.
Contradiction.

Picture of What is Going On



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So we have

$$\#_a(a^{m+j-i} b^m) \neq \#_b(a^{m+j-i} b^m) \implies a^{m+j-i} b^m \notin L_2.$$

Pumping Lemma

We Need a General Technique

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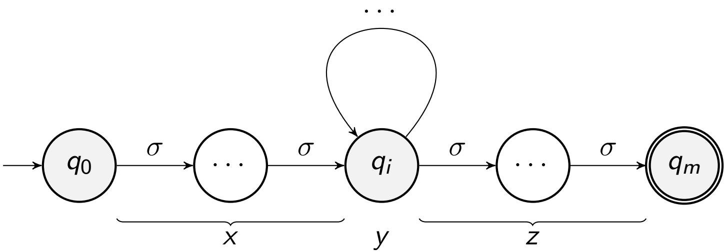
3. For all $i \geq 0$, $xy^i z \in L$.

Proof is picture on the next slide.

Proof by Pictures



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We then find some i such that $xy^iz \notin L$ for the contradiction.

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Contradiction since $m_2 \geq 1$.

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Proof: Same Proof as L_1 not Reg: Still look at $a^m b^m$.
Key Pumping Lemma says for ALL long enough $w \in L$.

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Go To Breakout Rooms To Work on it in Groups

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If L_3 is regular then $\overline{L_3} = L_2$ is regular. But we know that L_2 is not regular. DONE!

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By Pumping Lemma for long enough $a^{n^2} \in L_4$ there exists $x = a^{n_1}$, $y = a^{n_2}$, $z = a^{n_3}$ such that

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$$(n_1 + n_3) + in_2 \geq i^2$$

$$\frac{(n_1 + n_3)}{i} + n_2 \geq i$$

As i increases the LHS decreases and the RHS goes to infinity, so this cannot hold for all i.

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GOTO BREAKOUT ROOMS

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$$(n_1 + n_3) + (n_1 + n_2 + n_3 + 1)n_2 \text{ is a prime.}$$

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$$(n_1 + n_2 + n_3)(1 + n_2) \text{ is a prime.}$$

$L_6 = \{\#_a(w) > \#_b(w)\}$ is Not Regular

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Take $w = b^n a^{n+1}$, long enough so the y-part is in the b's.
Pump the y to get more b's than a's.

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Can **also** bound $|yz|$ by same proof.

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BREAKOUT ROOMS

Problematic Can take w long and pump a's, but that won't get out of the language.

So what to do? Revise Pumping Lemma

Pumping Lemma had a bound on $|xy|$.

Can **also** bound $|yz|$ by same proof.

Do that and then you can get y to be all b's, pump b's, and get out of the language.

$L_8 = \{a^{n_1} b^m c^{n_2} : n_1, n_2 > m\}$ is Not Regular

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For all i $xy^i z = a^{n_1 + in_2 + (n - n_1 - n_2)} b^{n-1} c^n \in L_8$.

Key We are used to thinking of i large. But we can also take $i = 0$, cut out that part of the word. We take $i = 0$ to get

$$xy^0 z = a^{n - n_2} b^{n-1} c^n$$

Since $n_2 \geq 1$, we have that $\#_a(xy^0 z) < n \leq n - 1 = \#_b(xy^0 z)$. Hence $xy^0 z \notin L_8$.

$i = 0$ Case as a Picture