BILL AND NATHAN RECORD LECTURE!!!!

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Regular Expressions

Recognizers vs Generators

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We want to write expressions that **generate** strings.

Regular Expressions over Σ

All the cool kids call them **regex**.

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Need to give examples and assign meaning.

Example and Meaning

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$a^* b \cup b^*$ is a regex. You can guess what it represents.
Def If α is a regex then $L(\alpha)$ is the set of strings it generates.

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Note that this is n not $O(n)$.

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4. Run the DFA M on a text to find where the pattern occurs.

Recap

We have

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Notation: $\delta(q, w)$

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$$\delta(q, e) = q.$$

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Dynamic Programming We will use all of this information to get our final answer.

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For all $1 \le i,j \le n$ $0 \le k \le n$, we will find a **regex** for $R(i,j,k)$.

Finding Regex for $R(i, j, k)$

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If a string goes from i to j with **no intermediary states** then it must just be a transition.

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Or $i = j$ and the string that is e.

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$$R(i,j,0) = \begin{cases} \{\sigma : \delta(i,\sigma) = j\} & \text{if } i \neq j\ \} \\ \{\sigma : \delta(i,\sigma) = j\} \cup \{e\} & \text{if } i = j\ \} \end{cases} \tag{1}$$

$R(i,j,0)$ is a Regex. Inductive Step

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We will now **assume** that for all $1 \leq i, j \leq n$, $R(i, j, k-1)$ is a Regex and **prove** that for all $1 \leq i, j \leq n$, $R(i, j, k)$ is a Regex.

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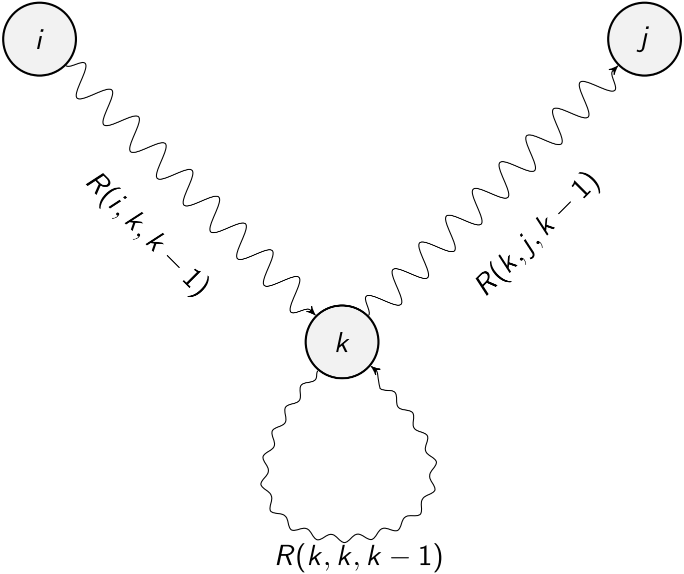
This is both of the following:

1. A proof by induction on k that, for all $1 \le i,j \le n$, $R(i,j,k)$ is a Regex.
2. A dynamic program that computes all $R(i,j,k)$.

Inductive Step $R(i, j, k)$ as a Picture



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For all $1 \leq i, j \leq n$ and all $0 \leq k \leq n$

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If ALL $R(i,j,k-1)$ are Regex, then ALL $R(i,j,k)$ are Regex.

Textbook Regular Expressions

Recall that lang $\{a, b\}^* a \{a, b\}^n$.

1. DFA requires 2^{n+1} states.
2. NFA can be done with $n + 2$ states.
3. How long is the regex for it? Regard the $\{a, b\}^* a$ part to be $O(1)$ length.

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there is a regex for L iff there is a trex for L.
A trex may give a much shorter expression than a regex.

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Can we show that every regex for L_n requires length $f(n)$ for some $f(n)$ where $\log n \ll f(n)$?

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Breakout Rooms!

Regex vs Trex For Length: Breakout Rooms!

Assume there is a regex for L_n of size $f(n)$ (we pick $f(n)$ later).

Regex vs Trex For Length: Breakout Rooms!

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Upshot There is a lang L_n with a trex of size $O(\log n)$ but the regex requires $\geq n$. Great! We have a large size difference.

Perl Regex and Java Regex

Regex and trex:

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1. **PRO** Can express many non-regular patterns such as L above.
2. **CON** The mathematical theory is not as clean. Maybe only people like me care.

BILL AND NATHAN STOP RECORDING LECTURE!!!!

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