Tricks for Divisibility and DFA’s
What I Learned in School

Divisibility tricks for

- Even/odd
- 10
- 5
- 3
- 9
- 11

What is a trick?
What I Learned in School

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What is a trick?
For this Slide Packet $\Sigma = \{0, \ldots, 9\}$.
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Strings are numbers in base 10. The string

$$d_{n-1} \cdots d_0$$

is the number

$$d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10^1 + d_0 \times 10^0.$$
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We feed a number into a DFA right-to-left: $d_0$, then $d_1$ etc.
Did you know? A number is even iff its last digit is even.
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**Thm** \( d_{n-1} \cdots d_0 \equiv d_0 \pmod{2} \).
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**Thm** $d_{n-1} \cdots d_0 \equiv d_0 \pmod{2}$.

**Pf**

$$d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0$$
Did you know? A number is even iff its last digit is even. We state this a different way so can generalize later.

**Thm** \( d_{n-1} \cdots d_0 \equiv d_0 \pmod{2} \).

**Pf**

\[
\begin{align*}
d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0 \\
= 10(d_{n-1} \times 10^{n-2} + \cdots + d_1) + d_0
\end{align*}
\]
**Did you know?** A number is even iff its last digit is even. We state this a different way so can generalize later.

**Thm** $d_{n-1} \cdots d_0 \equiv d_0 \pmod{2}$.

**Pf**

$$d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0$$

$$= 10(d_{n-1} \times 10^{n-2} + \cdots + d_1) + d_0$$

$$\equiv d_0 \pmod{2}$$
DFA for Even
DFA for Even

```
1, 3, 5, 7, 9
```

```
0, 2, 4, 6, 8
```

```
0
```

```
1
```

```
* 
```
Trick for Divisible by 3

Did you Know? A number is divisible by 3 iff sum of digits is divisible by 3.
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We state this a different way which gives more information.

Thm \( d_{n-1} \cdots d_0 \equiv d_{n-1} + \cdots + d_0 \pmod{3}. \)

Pf

\[
d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0 \times 10^0
\]
Trick for Divisible by 3

Did you Know? A number is divisible by 3 iff sum of digits is divisible by 3.
We state this a different way which gives more information.

Thm $d_{n-1} \cdots d_0 \equiv d_{n-1} + \cdots + d_0 \pmod{3}$.

Pf

\[
\begin{align*}
    d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0 \times 10^0 \\
\equiv \quad d_{n-1} \times 1 + \cdots + d_1 \times 1 + d_0 \times 1 \pmod{3}
\end{align*}
\]
Did you Know? A number is divisible by 3 iff sum of digits is divisible by 3.

We state this a different way which gives more information.

**Thm** $d_{n-1} \cdots d_0 \equiv d_{n-1} + \cdots + d_0 \pmod{3}$.

**Pf**

\[
d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0 \times 10^0 \\
\equiv d_{n-1} \times 1 + \cdots + d_1 \times 1 + d_0 \times 1 \pmod{3} \\
\equiv d_{n-1} + \cdots + d_1 + d_0 \pmod{3}
\]
DFA for Divisible by 3
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Trick for Mod 4. All $\equiv$ are Mod 4

Did you Know? $n \equiv 0$ iff
Trick for Mod 4. All $\equiv$ are Mod 4

**Did you Know?** $n \equiv 0$ iff last 2 digits are a number $\equiv 0$. 
Trick for Mod 4. All $\equiv$ are Mod 4

Did you Know? $n \equiv 0$ iff last 2 digits are a number $\equiv 0$.

Thm $d_{n-1} \cdots d_0 \equiv 2d_1 + d_0$. 
Trick for Mod 4. All \( \equiv 0 \) are Mod 4

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Pf

\[
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Pf

\[
d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0
\equiv d_1 \times 10 + d_0
\]
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Thm \( d_{n-1} \cdots d_0 \equiv 2d_1 + d_0 \).

Pf

\[
d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0 \\
\equiv d_1 \times 10 + d_0 \\
\equiv 2d_1 + d_0.
\]
DFA for Divisible by 4
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- States: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- Transitions:
  - From 0: 0, 4, 8
  - From 1: 1, 3, 5, 7, 9
  - From 2: 2, 6
  - From 3: 0, 2, 4, 6, 8
  - From 4: 1, 3, 5, 7, 9
  - From 5: *
  - From 6: *
  - From 7: *
  - From 8: *
  - From 9: *
Key to all of these Problems

For all of these problems we need to find a pattern of $10^n \pmod{a}$. 
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Mod 2: Pattern is 1,0,0,0,\ldots, DFA only cared about first digit.
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Mod 2: Pattern is 1,0,0,0,…, DFA only cared about first digit.
Mod 3: Pattern is 1,1,1,1,…, DFA tracked sum of digits mod 3.
Key to all of these Problems

For all of these problems we need to find a pattern of $10^n \pmod{a}$.
Mod 2: Pattern is 1,0,0,0,\ldots, DFA only cared about first digit.
Mod 3: Pattern is 1,1,1,1,\ldots, DFA tracked sum of digits mod 3.
Mod 4: Pattern is 1,2,0,0,0,\ldots, DFA only cared about first 2 digits.
Tricks for Mod 5 and Mod 6

These may be on a HW.
Trick for Mod 11. All \(\equiv\) are Mod 11

Is there a trick for mod 11?

Did you Know? \(n \equiv 0\) iff \(\pm\) sum of digits is \(\equiv 0\).
Trick for Mod 11. All $\equiv$ are Mod 11

Is there a trick for mod 11?

Did you Know? $n \equiv 0$ iff $\pm$ sum of digits is $\equiv 0$.

Thm $d_n \cdots d_0 \equiv d_0 - d_1 + d_2 - \cdots \pm d_n$. 
Is there a trick for mod 11?

**Did you Know?** $n \equiv 0$ iff $\pm$ sum of digits is $\equiv 0$.

**Thm** $d_n \cdots d_0 \equiv d_0 - d_1 + d_2 - \cdots \pm d_n$.

Proof may be on HW or Midterm or Final or some combination.
DFA for Mod 11

Need to keep track of both the running weighted sum mod 11 and if you are reading an even or odd place.
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Need to keep track of both the running weighted sum mod 11 and if you are reading an even or odd place.

\[ Q = \{0, \ldots, 10\} \times \{0, 1\} \]
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\[ Q = \{0, \ldots, 10\} \times \{0, 1\} \]

\[ s = (0, 0). \]
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Final state: Not going to have these, this is DFA-classifier.
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\[
\delta((i, j), \sigma) \begin{cases} 
(i + \sigma \pmod{11}, j + 1 \pmod{2}) & \text{if } j = 0 \\
(i - \sigma \pmod{11}, j + 1 \pmod{2}) & \text{if } j = 1 
\end{cases}
\]  

(1)
DFA for Mod 11

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(1)

We keep track of a running weighted sum mod 11 and position of the symbol mod 2.
DFA for Mod 11

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22 states.
DFA for Mod 11

Need to keep track of both the running weighted sum mod 11 and if you are reading an even or odd place.

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Final state: Not going to have these, this is DFA-classifier.

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\end{cases}
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(1)

We keep track of a running weighted sum mod 11 and position of the symbol mod 2.

22 states.

**Classifier** If end in \((i, 0)\) or \((i, 1)\) then number is \(\equiv i\).
Is there a trick for mod 7?

\[\begin{align*}
10^0 & \equiv 1 \\
10^1 & \equiv 3 \\
10^2 & \equiv 10 \times 10 \equiv 3 \times 3 \equiv 9 \equiv 2 \\
10^3 & \equiv 10^2 \times 10 \equiv 2 \times 3 \equiv 6 \\
10^4 & \equiv 10^3 \times 10 \equiv 6 \times 3 \equiv 18 \equiv 4 \\
10^5 & \equiv 10^4 \times 10 \equiv 4 \times 3 \equiv 12 \equiv 5 \\
10^6 & \equiv 10^5 \times 10 \equiv 5 \times 3 \equiv 15 \equiv 1 \\
\end{align*}\]

Pattern is 1, 3, 2, 6, 4, 5, 1, ...
Is There a Mod 7 Trick? \( \equiv \) is Mod 7

Is there a trick for mod 7?

**Answer** Depends what you call a trick.

We need to spot a pattern.

\[
egin{align*}
10^0 & \equiv 1 \\
10^1 & \equiv 3 \\
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10^3 & \equiv 5 \\
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10^5 & \equiv 3 \\
10^6 & \equiv 2 \\
10^7 & \equiv 6 \\
10^8 & \equiv 4 \\
10^9 & \equiv 5 \\
10^{10} & \equiv 1 \\
10^{11} & \equiv 3 \\
10^{12} & \equiv 2 \\
10^{13} & \equiv 6 \\
10^{14} & \equiv 4 \\
10^{15} & \equiv 5 \\
10^{16} & \equiv 1 \\
\end{align*}
\]

Pattern is 1, 3, 2, 6, 4, 5, 1, \ldots.

Can we use this?
Is there a trick for mod 7?

**Answer** Depends what you call a trick.
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Is There a Mod 7 Trick? \equiv is Mod 7

Is there a trick for mod 7?

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$10^0 \equiv 1$
Is there a trick for mod 7?

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\[10^0 \equiv 1\]
\[10^1 \equiv 3\]
Is there a trick for mod 7?

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\begin{align*}
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\]

Pattern is 1, 3, 2, 6, 4, 5, 1, 3, 2, 6, 4, 5, 1, ...
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\end{align*}
\]

Pattern is 1, 3, 2, 6, 4, 5, 1, 3, 2, 6, 4, 5, 1, ... 

Can we use this?
Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.
Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.

\[
3876554
= 3 \cdot 10^6 + 8 \cdot 10^5 + 7 \cdot 10^4 + 6 \cdot 10^3 + 5 \cdot 10^2 + 5 \cdot 10 + 4
\]
Using the Divide by 7 Trick

Want to know what $3876554$ is mod 7.

$$3876554$$
$$\equiv 3 \cdot 10^6 + 8 \cdot 10^5 + 7 \cdot 10^4 + 6 \cdot 10^3 + 5 \cdot 10^2 + 5 \cdot 10 + 4$$
$$\equiv 3 \cdot 1 + 8 \cdot 5 + 7 \cdot 4 + 6 \cdot 6 + 5 \cdot 2 + 5 \cdot 3 + 4 \pmod{7}$$
Using the Divide by 7 Trick

Want to know what $3876554$ is mod 7.

$$3876554$$

$$= 3 \cdot 10^6 + 8 \cdot 10^5 + 7 \cdot 10^4 + 6 \cdot 10^3 + 5 \cdot 10^2 + 5 \cdot 10 + 4$$

$$\equiv 3 \cdot 1 + 8 \cdot 5 + 7 \cdot 4 + 6 \cdot 6 + 5 \cdot 2 + 5 \cdot 3 + 4 \pmod{7}$$

$$\equiv 3 \cdot 1 + 1 \cdot 5 + 0 \cdot 4 + -1 \cdot 6 + -2 \cdot 2 + -2 \cdot 3 + 4 \pmod{7}$$
Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.

\[ 3876554 \equiv 3 \cdot 1 + 8 \cdot 5 + 7 \cdot 4 + 6 \cdot 6 + 5 \cdot 2 + 5 \cdot 3 + 4 \pmod{7} \]
\[ \equiv 3 + 1 + 5 + 0 - 1 - 2 - 2 + 3 + 4 \pmod{7} \]
\[ \equiv 3 + 0 - 4 - 6 + 4 \pmod{7} \]
Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.

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\begin{align*}
3876554 & \equiv 3 \cdot 10^6 + 8 \cdot 10^5 + 7 \cdot 10^4 + 6 \cdot 10^3 + 5 \cdot 10^2 + 5 \cdot 10 + 4 \\
& \equiv 3 \cdot 1 + 8 \cdot 5 + 7 \cdot 4 + 6 \cdot 6 + 5 \cdot 2 + 5 \cdot 3 + 4 \pmod{7} \\
& \equiv 3 \cdot 1 + 1 \cdot 5 + 0 \cdot 4 + -1 \cdot 6 + -2 \cdot 2 + -2 \cdot 3 + 4 \pmod{7} \\
& \equiv 3 + 5 + 0 - 6 - 4 - 6 + 4 \pmod{7} \\
& \equiv 3 \pmod{7}
\end{align*}
\]
Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.

\[ 3876554 \equiv 3 \cdot 1 + 8 \cdot 5 + 7 \cdot 4 + 6 \cdot 6 + 5 \cdot 2 + 5 \cdot 3 + 4 \pmod{7} \]
\[ \equiv 3 + 5 + 0 - 6 - 4 - 6 + 4 \pmod{7} \]
\[ \equiv 3 \pmod{7} \]

**DFA** States will keep track of
Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.

\[
3876554 \\
≡ 3 \cdot 10^6 + 8 \cdot 10^5 + 7 \cdot 10^4 + 6 \cdot 10^3 + 5 \cdot 10^2 + 5 \cdot 10 + 4
\]

\[
≡ 3 \cdot 1 + 8 \cdot 5 + 7 \cdot 4 + 6 \cdot 6 + 5 \cdot 2 + 5 \cdot 3 + 4 \pmod{7}
\]

\[
≡ 3 \cdot 1 + 1 \cdot 5 + 0 \cdot 4 + (−1) \cdot 6 + (−2) \cdot 2 + (−2) \cdot 3 + 4 \pmod{7}
\]

\[
≡ 3 + 5 + 0 − 6 − 4 − 6 + 4 \pmod{7}
\]

\[
≡ 3 \pmod{7}
\]

**DFA** States will keep track of
Running weighted sum mod 7
Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.

\[ 3876554 \equiv 3 \cdot 1 + 8 \cdot 5 + 7 \cdot 4 + 6 \cdot 6 + 5 \cdot 2 + 5 \cdot 3 + 4 \pmod{7} \]
\[ \equiv 3 + 5 + 0 - 6 - 4 - 6 + 4 \pmod{7} \]
\[ \equiv 3 \pmod{7} \]

**DFA** States will keep track of
Running weighted sum mod 7
Position of digit mod 6 so know which weights to use.
Using the Divide by 7 Trick

Want to know what $3876554$ is mod 7.

\[
3876554 \\
\equiv 3 \cdot 10^6 + 8 \cdot 10^5 + 7 \cdot 10^4 + 6 \cdot 10^3 + 5 \cdot 10^2 + 5 \cdot 10 + 4 \\
\equiv 3 \cdot 1 + 8 \cdot 5 + 7 \cdot 4 + 6 \cdot 6 + 5 \cdot 2 + 5 \cdot 3 + 4 \pmod{7} \\
\equiv 3 \cdot 1 + 1 \cdot 5 + 0 \cdot 4 + -1 \cdot 6 + -2 \cdot 2 + -2 \cdot 3 + 4 \pmod{7} \\
\equiv 3 + 5 + 0 - 6 - 4 - 6 + 4 \pmod{7} \\
\equiv 3 \pmod{7}
\]

**DFA** States will keep track of Running weighted sum mod 7
Position of digit mod 6 so know which weights to use. So there are $7 \times 6 = 42$ states.
Is the Method a Trick?

YES

A DFA can do it.

NO

A human cannot do it easily. (The pattern is not like \(1,1,1,\ldots\) or mostly 0's.)
Is the Method a Trick?

**YES** A DFA can do it.
Is the Method a Trick?

YES A DFA can do it.

NO A human cannot do it easily. (The pattern is not like 1,1,1,\ldots or mostly 0’s.)
The DFA for \( \{n : n \equiv 0 \pmod{7}\} \)
The DFA for \( \{ n : n \equiv 0 \pmod{7} \} \)

Too hard for me ...
The DFA for \{ n : n \equiv 0 \pmod{7} \}

Too hard for me ...

... but not for you.
The DFA for \( \{ n : n \equiv 0 \pmod{7} \} \)

Too hard for me ...

... but not for you.

Might make it a HW to do as a table.
Possible Research Question

What is the fastest way to determine \( n \pmod{7} \)?
What is the fastest way to determine \( n \pmod{7} \)?

**Method One** Divide and take remainder.
Possible Research Question

What is the fastest way to determine $n \pmod{7}$?

**Method One** Divide and take remainder.

**Method Two** Use the DFA.
Possible Research Question

What is the fastest way to determine $n \pmod{7}$?

**Method One** Divide and take remainder.

**Method Two** Use the DFA.

**Question** Which is faster?
What is the fastest way to determine $n \pmod{7}$?

**Method One** Divide and take remainder.

**Method Two** Use the DFA.

**Question** Which is faster?

Might be hard to tell because today’s computers are so fast!