Help On the Classification HW

Classification Problem

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1) Find the pattern of
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$\vdots$
(You should write a program to help you with this.)

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(You should write a program to help you with this.)

2) Find the size of a DFA to classify mod n.

We Do The Problem For $n = 7$

All $\equiv$ is mod 7.

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The DFA to classify mod 7 has to keep track of
1) The weighted sum mod 7.
2) The position of the digit mod 6.
So the number of states is $7 \times 6 = 42$.

DFA-Classifier for Mod 7

(This is not required for the HW.)
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$\delta((q, 4), \sigma) = (q + 4 \times \sigma \pmod 7, 5)$

$\delta((q, 5), \sigma) = (q + 5 \times \sigma \pmod 7, 0)$

Another Example

Mod 37

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Pattern is $(1, 10, 26)$.

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Pattern length 3, mod is 37, so DFA has $37 \times 3 = 111$ states.

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Drawing the DFA would only be 3 cases.

Pattern $\overline{a_1, \ldots, a_m}$

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So $Q = \{0, \ldots, n-1\} \times \{0, \ldots, m-1\}$.

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The number of states is nm.

What If The patterns might not be of this form?

Pattern $(a_1, \ldots, a_{m-1}, \overline{a_m})$

Mod 6.

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Pattern is $(1, \overline{4})$.

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So have the pattern. How big is the DFA?

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Pattern is $(1, \overline{4})$.
So have the pattern. How big is the DFA?
In groups try to design the DFA.

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$\delta(s, \sigma) = \quad 1 \times \sigma \pmod 6$

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If $q \in \{0, 1, 2, 3, 4, 5\}$, $\delta(q, \sigma) = q + 4\sigma \pmod 6$
Number of states: $1 + 6 = 7$.

What if Sequence is 2 Terms Then Repeats

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Mod 375.

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Mod 375. Pattern is $(1, 10, 100, \overline{250})$.

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Edges out of the start state have weight 1.

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Mod 375. Pattern is $(1, 10, 100, \overline{250})$.

Edges out of the start state have weight 1.

Edges out of the second set of states have weight 10.

What if Sequence is 2 Terms Then Repeats

Mod 375. Pattern is $(1, 10, 100, \overline{250})$.

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Edges out of the second set of states have weight 10.

Edges out of the third set of states have weight 100.

What if Sequence is 2 Terms Then Repeats

Mod 375. Pattern is $(1, 10, 100, \overline{250})$.

Edges out of the start state have weight 1.

Edges out of the second set of states have weight 10.

Edges out of the third set of states have weight 100.

Edges out of the fourth set of states have weight 250 and only go to the fourth set of states.

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$Q = \{s\} \cup \{0, \ldots, 3742\} \times \{1, 2, 3\}$
$\delta(s, \sigma) = \quad (1 \times \sigma \pmod{375}, 1)$.

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Leave it to you to work this out.

Pattern $(a_1, \ldots, a_{m-1}, \overline{0})$

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Identical to previous slides. The DFA is easier since the last set of states have transition to themselves.