On the Sizes of Descriptions of Languages

William Gasarch-U of MD

Devices for Langauges

Devices of interest: DFA, NDFA, CFG.

Definition: f is a *bounding function for* (DFA,CFG) if for all n, for all CFLs A that have a CFG of size n, if A is also REG then A has a DFA of size $\leq f(n)$. (Similar for (DFA,NDFA).)

Examples:

$f(n) = 2^n$ is a bounding function for (DFA,NDFA)

Bounding Function for (DFA,NDFA)

For all n let $L_n = \{a, b\}^* a \{a, b\}^n$. (S

- There is an NDFA for L_n of size $O(n)$.
- Any DFA for L_n is of size $\geq 2^n$.
- There is a DFA for L_n of size 2^n.

Corollary

If f is a bounding function for (DFA,NDFA) then

$$(\forall n)[f(n) \geq 2^{\Omega(n)}].$$

Bounding Function for (NDFA,CFG)

For all n let $L_n = \{a^{2^n}\}$.

- There is an CFG for L_n of size $O(n)$.
- Any NDFA for L_n is of size $\geq 2^n$.
- There is an NDFA for L_n of size 2^n.

Corollary

If f is a bounding function for (NDFA,CFG) then

$$(\forall n)[f(n) \geq 2^{\Omega(n)}].$$

Bounding Function for (DFA,CFG)- Configs

Let $e, x \in N$. $ACC_{e,x}$ is the set of all

$$\$C_1\$C_2^R\$C_3\$C_4^R\$\cdots\$C_s^R\$$$

such that

- $|C_1| = |C_2| = \cdots = |C_s|$.
- $C_1, C_2, \ldots, C_s$ is the accepting computation of $M_e(x)$.

Note that:

- $\overline{ACC_{e,x}} \in L(CFG)$ (use PDAs).
- Computable to go from e, x to $ACC_{e,x}$.
- $M_e(x) \downarrow \implies |ACC_{e,x}| = \{\text{acc config seq for } M_e(x) \downarrow\}$.
- $M_e(x) \uparrow \implies |ACC_{e,x}| = \emptyset$.

HALT $\le_T$ Bounding Function for (DFA,CFG)

(Hartmanis)
Let f be a bounding function for (DFA,CFG).
$HALT \le_T f$.

1. Input(e, x).
2. Create CFG for $\overline{ACC_{e,x}}$. Let n be its size. Let $t = f(n)$.
   Note: DFA for $\overline{ACC_{e,x}} \le t \implies$ DFA for $ACC_{e,x} \le t$.
3. For all DFA A of size $\le t$
   3.1 Check if $L(A) \ne \emptyset$.
   3.2 If so then find a $w \in L(A)$.
   3.3 If w is acc comp of $M_e(x) \downarrow$ then output YES and stop
4. Output(NO).

Bounding Function for (DFA,CFG)

Corollary: If f is a bounding function for (DFA,CFG) then $HALT \leq_{\mathrm{T}} f$

Thoughts:

Is there a bdd fn for (DFA,CFG) of Turing Degree HALT?

VOTE

1. YES
2. NO
3. UNKNOWN TO SCIENCE

Bounding Function for (DFA,CFG)

Corollary: If f is a bounding function for (DFA,CFG) then $HALT \leq_{\mathrm{T}} f$

Thoughts:

Is there a bdd fn for (DFA,CFG) of Turing Degree HALT?

VOTE

1. YES
2. NO
3. UNKNOWN TO SCIENCE

- In Aug 2016 it would be UNKNOWN TO SCIENCE.
- In Sep 2016 **Gasarch** proved NO. Bdd fn exactly Σ_2.