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They are better than PRG's!

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We will not do that.

However,

we are right, and they are wrong.

Stream ciphers

- As we defined them, PRGs are limited
  - They have fixed-length output
  - They produce output in "one shot"
- In practice, PRGs are based on *stream ciphers*
  - Can be viewed as producing an "infinite" stream of pseudorandom bits, on demand
  - More flexible, more efficient

Stream ciphers

A Stream Cipher is a pair of efficient, deterministic algorithms (Init, GetBits) such that:

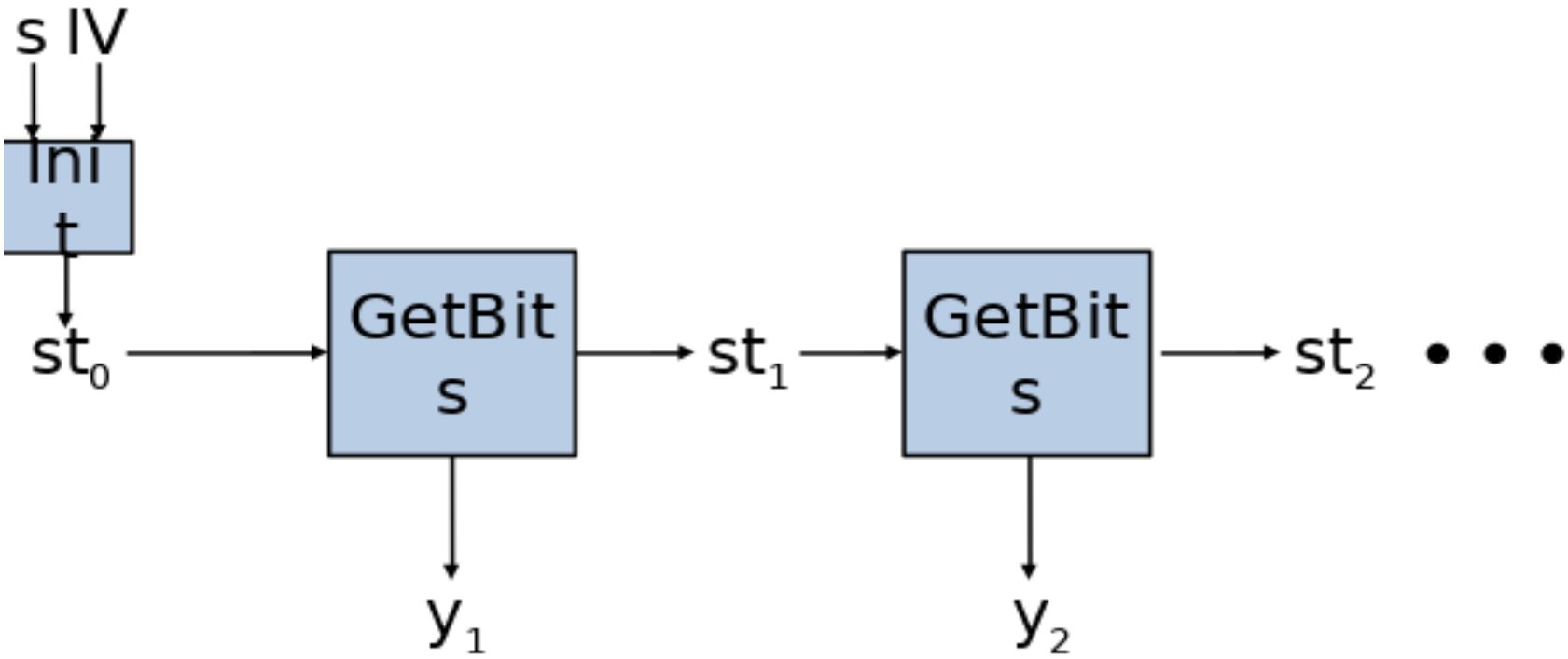
1. Init does the following:
   - 1.1 Input: private seed s, opt public Init Vector (IV) V
   - 1.2 Output: initial state st_0

2. GetBits does the following:
   - 2.1 Input: current state st
   - 2.2 Output: a bit y along with updated state st'

Note In practice, y is a block rather than a bit.

Stream ciphers

- Can use (Init, GetBits) to generate any desired number of output bits from an initial seed



Stream ciphers

- A stream cipher is *secure* (informally) if the output stream generated from a uniform seed is pseudorandom
  - I.e. regardless of how long the output stream is (so long as it is polynomial)
  - See book for formal definition

Do Stream Ciphers exist? Theoretical

A one-way function (perm) is function (perm): easy to compute, hard to invert.
A one-way function (perm) with a hard core predicate is a function (perm) that is easy to compute but hard to invert, and (say) the middle bit of $f^{-1}(x)$ is hard to compute.

Chapter 7 shows:
$\exists$ One way Perm $\Longrightarrow$ $\exists$ one way perm with a hcp.
$\exists$ one way perm with hcp $\Longrightarrow$ $\exists$ PRG with expansion 1
$\exists$ PRG with expa-1 $\Longrightarrow$ $\exists$ Stream Ciphers
Note: (1-way func $\Longrightarrow$ $\exists$ SC's) known but much harder.
Note: Stream Cipher obtained this way too slow to use :-(
Note: Proof of concept valuable :-)

Do Stream Ciphers exist? Practical

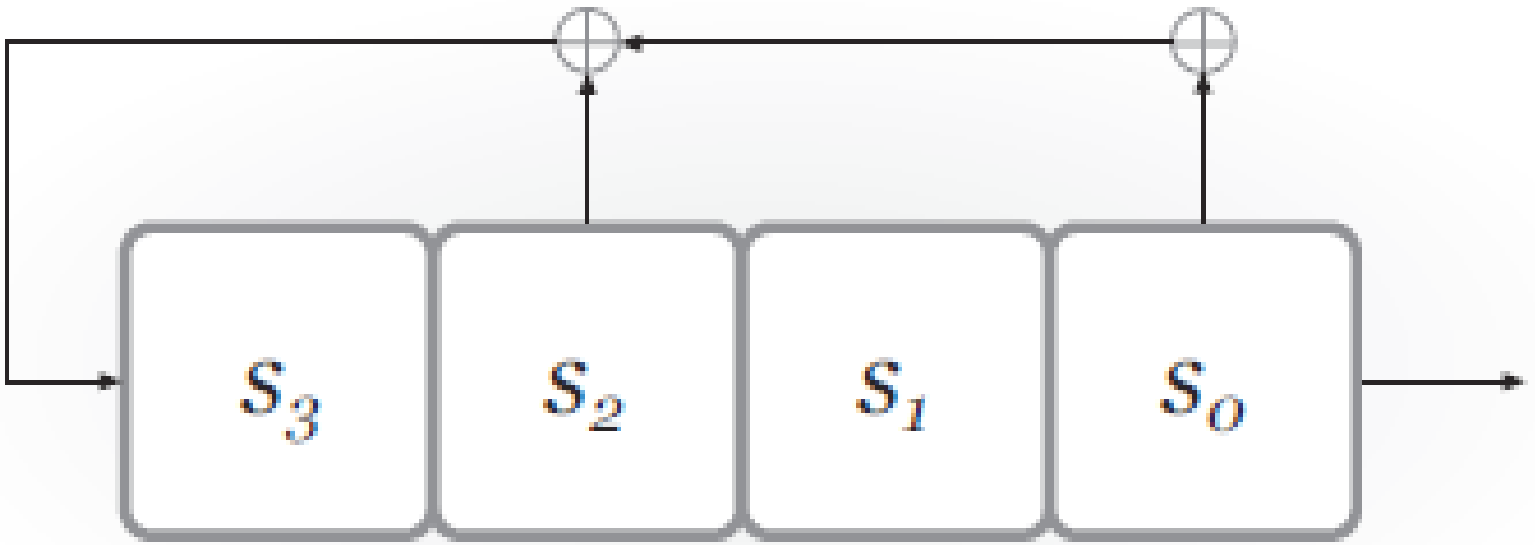
Several attempt Stream Ciphers:

1. Linear Feedback Shift Registers. Fast! Used! Not Secure!
2. Trivium. Fast! Used! Empirically Secure?
3. Rivest Cipher 4. Fast! Used! No longer secure!

Note Seems impossible to get Stream Ciphers that are provably (even using Hardness Assumptions) secure and practical.

Note But having the rigor gives the practitioners (1) a target to shoot for, and (2) pitfalls to watch out for.

Example of Linear Feedback Shift Register



- Assume initial content of registers is 0100
- First 4 state transition: $0100 \rightarrow 1010 \rightarrow 0101 \rightarrow 0010 \rightarrow \ldots$
- First 3 output bits: $001\ldots$

Linear Feedback Shift Registers (LFSR): Example

Degree 3 LFSR, 3 constants : $c_2, c_1, c_0 \in \{0, 1\}$. $+$ is mod 2.

Key is st_0 is 3 bits: (s_2^0, s_1^0, s_0^0). NO IV (for now).

$$st_1 = (s_2^1, s_1^1, s_0^1) = (c_2 s_2^0 + c_1 s_1^0 + c_0 s_0^0, s_2^0, s_1^0).$$

$$st_{t+1} = (s_2^t, s_1^t, s_0^t) = (c_2 s_2^t + c_1 s_1^t + c_0 s_0^t, s_2^t, s_1^t).$$

In English: Bits shift right, left most bit is c-combo of prior bits.

$$y_1 = s_0^0 \qquad y_2 = s_1^0 \qquad y_3 = s_2^0$$

$$y_4 = s_2^1 = c_2 y_3 + c_1 y_2 + c_0 y_1$$

$$y_t = s_2^{t-3} = c_2 y_{t-3} + c_1 y_{t-2} + c_0 y_{t-1}$$

In English: y_t is (1) left most bit of st_{t-3} & (2) c-combo of prior y.

Note the Two Definitions of y_t

$$y_t = s_3^{t-3} = c_2 y_{t-3} + c_1 y_{t-2} + c_0 y_{t-1}$$

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Linear Feedback Shift Registers (LFSR)

Degree n LFSR, n constants : $c_{n-1}, \ldots, c_0 \in \{0, 1\}$. $+$ is mod 2.

Key is st_0 is n bits: $(s^0_{n-1}, \ldots, s^0_0)$. NO IV (for now).

$$st_1 = (s^1_{n-1}, \ldots, s^1_0) = (c_{n-1}s^0_{n-1} + \cdots + c_0 s^0_0, s^0_{n-1}, s^0_{n-3}, \ldots, s^0_1).$$

$$st_{t+1} = (s^t_{n-1}, \ldots, s^t_0) = (c_{n-1}s^t_{n-1} + \cdots + c_0 s^t_0, s^t_{n-1}, s_{n-2}, \ldots, s^t_1).$$

In English: Bits shift right, left most bit is c-combo of prior bits.

$$y_1 = s^0_0 \qquad \cdots \qquad y_n = s^0_{n-1}$$

$$y_n = s^1_n = c_{n-1}y_n + \cdots + c_0 y_1$$

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LFSR with IV

LFSR of degree n is defined by $c_{n-1}, \ldots, c_0$ all in $\{0, 1\}$

Key is st'_0 is n bits. IV is n bits IV. $st_0 = st'_0 \oplus IV$.
All the rest is the same.
In English: XOR the private key with the public IV.
Why do this? Next Slide.

Two Ways to Use Stream Ciphers

Two Ways to Use Stream Ciphers. We illustrate with LFSR.

1. Syn Mode Alice gen and send private key. Bob sends message of length L, using $y_1 \cdots y_L$. Bob uses key to get $y_1 \cdots y_L$ and decode. Bob responds with message of length M using $y_{L+1} \cdots y_{L+M}$. They both keep in sync.
2. Unsyn Mode Alice gen and send private key. Alice sends public IV. Bob sends message of length L AND IV. Alice uses key and IV to get $y_1 \cdots y_L$ and can decode. Alice sends message of length M AND IV. Bob uses Key and IV to get $y_1 \cdots y_M$ and can decode. They do not have to be in sync.

When to use which? Discuss

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When to use which? Discuss

1. Use Sync if all communication will be in one session and communication is clear.
2. Use Unsync if Alice talks Monday, Bob replies Tuesday perhaps on a diff device or if Comm is noisy.

LFSRs as stream ciphers

- Key + IV used to initialize the state of the LFSR
- Every clock tick:
  1. State Updated
  2. Bit output

LFSRs

1. State (and output) "cycles" if state ever repeated
2. *Maximal-length LFSR* cycles through all $2^n - 1$ nonzero states
3. Known how to set feedback coefficients so as to achieve maximal length
4. Maximal-length LFSRs have good statistical properties . . .
5. Are LFSRs secure? Vote YES, NO, UNKNOWN TO SCIENCE.

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Example of Bad Security

Degree 3. c_0, c_1, c_2 unknown. s_0^0, s_1^0, s_2^0 unknown.

$y_1 = s_0^0$
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$$y_4 = c_2 y_3 + c_1 y_2 + c_0 y_1$$
$$y_5 = c_2 y_4 + c_1 y_3 + c_0 y_2$$
$$y_6 = c_2 y_5 + c_1 y_4 + c_0 y_3$$

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3 linear equations in 3 variables. Can find c_0, c_1, c_2. Cracked!

For n-degree LFSR can crack after $2n$ iterations.
Moral: Linearity is *bad* cryptography.

LFSR and Linearity

Linearity makes LFSR's fast

Linearity makes LFSR's crackable

LFSR and Linearity

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Its that old saying:

He who lives by linearity, dies by linearity.

The Essence of Crypto

Recall: The Essence of Crypto is to make computation

1. Easy for Alice and Bob.
2. Hard for Eve.

LFSR makes computation easy for all three!

Nonlinear Feedback Shift Registers (FSRs)

- Add nonlinearity to prevent attacks
  - Nonlinear feedback
  - Output is a nonlinear function of the state
  - Multiple (coupled) LFSRs
  - . . . or any combination of the above
- Still want to preserve statistical properties of the output, and long cycle length

Nonlinear Feedback Shift Registers

Assume n even. $+$ is mod 2.
Let $f(x_1, \ldots, x_n) = x_1x_2 + x_3x_4 + \cdots x_{n-1}x_n$.
st_0 is n bits: $(s^0_{n-1}, \ldots, s^0_0)$.

For $i = 1$ to ∞

$$st_i = (s^i_{n-1}, \ldots, s^i_0) = (f(s^{i-1}_{n-1}, \ldots, s^{i-1}_0), s^{i-1}_{n-1}, s^{i-1}_{n-2}, \ldots, s^{i-1}_1)$$

$$y_i = s^i_0$$

In English: Bits shift right, left bit is f of bits at last stage.
Is this a good stream cipher? Vote Y (with HA), N, UN

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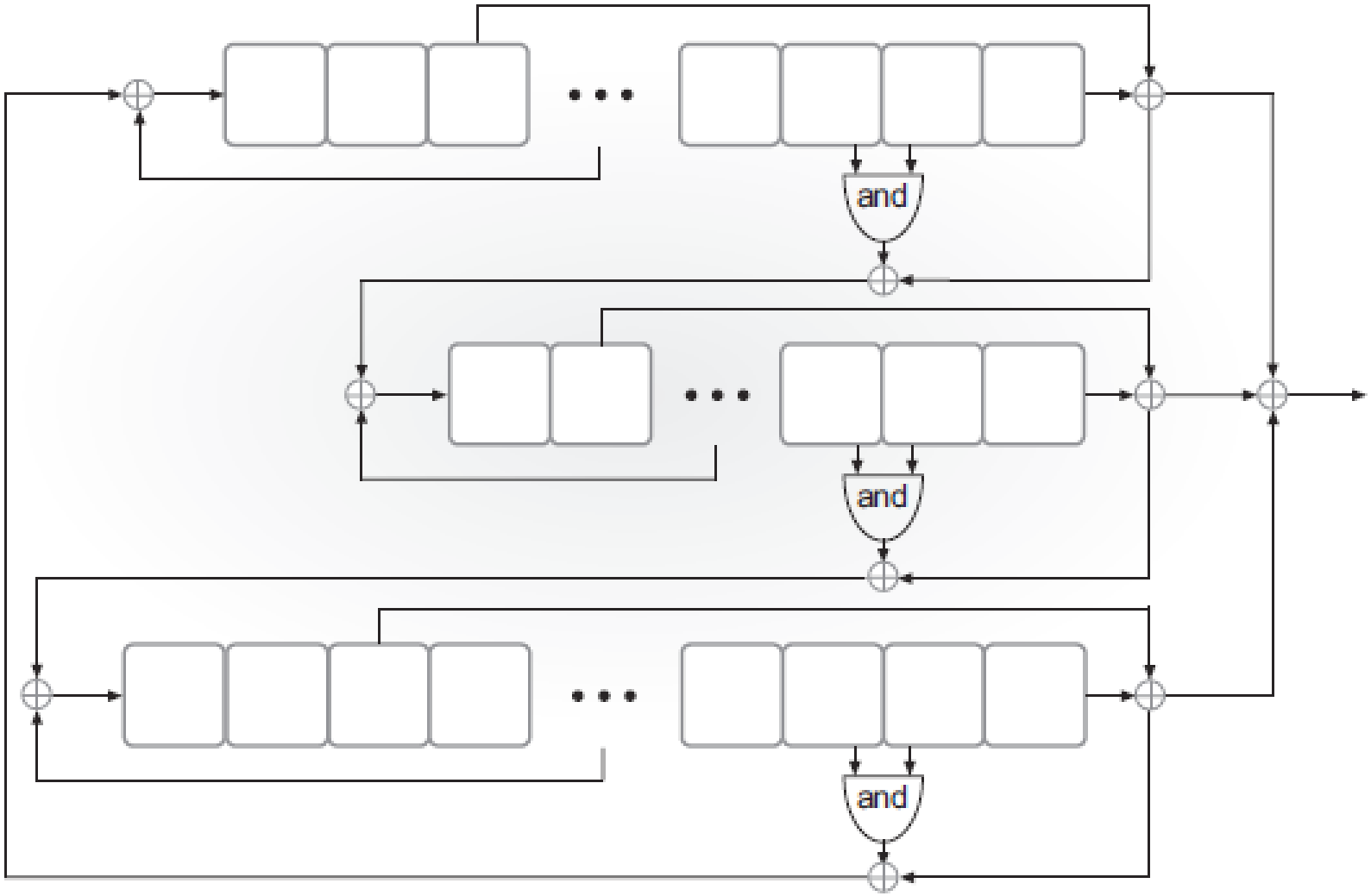
UN: I made up this cipher last month for example of nonlinear.

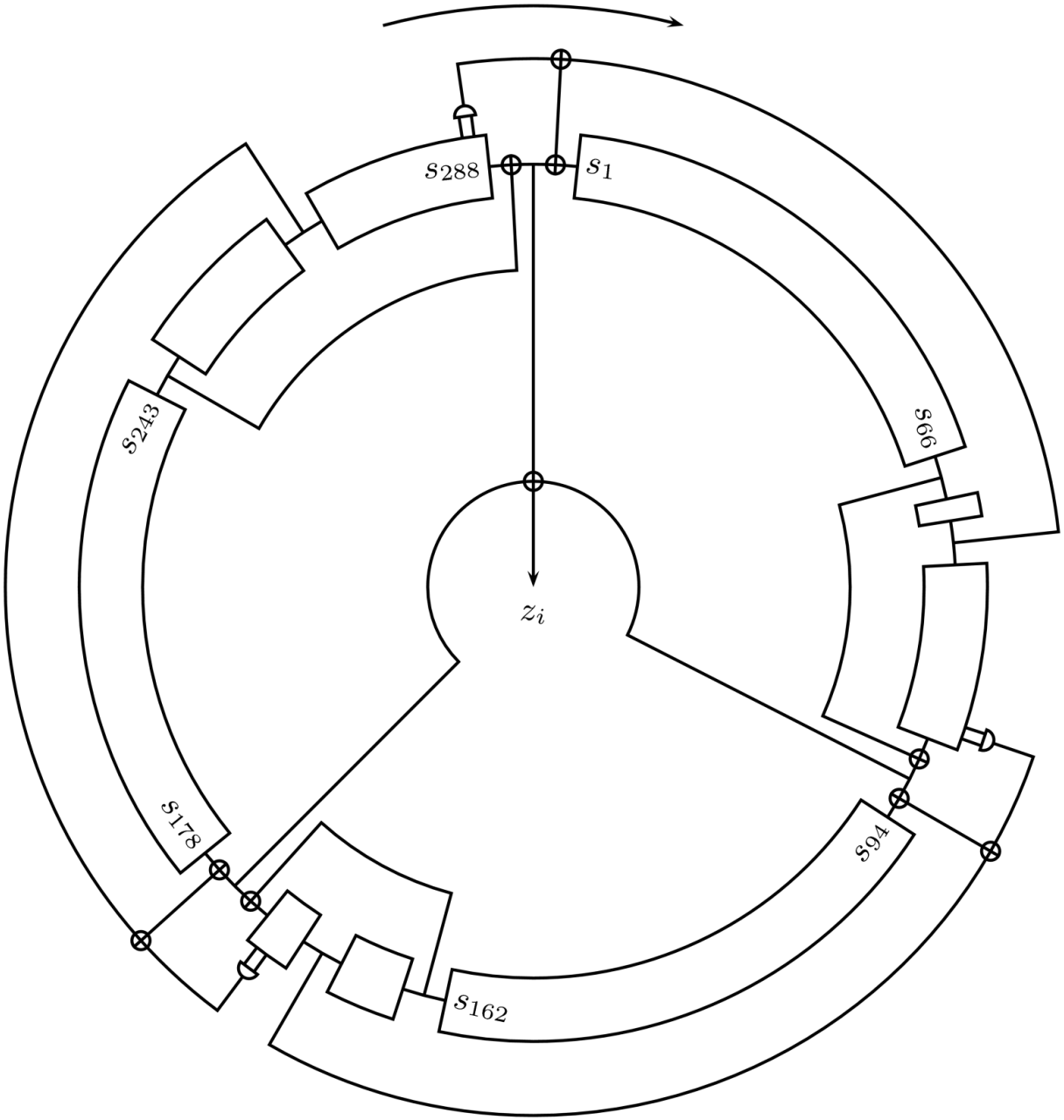
Trivium

- Designed by De Cannière and Preneel in 2006 as part of eSTREAM competition
- Intended to be simple and efficient (especially in hardware)
- Essentially no attacks better than brute-force search are known

Trivium Hardware Abstractly



Trivium Hardware For Real



Trivium

- Three coupled Feedback Shift Regisers (FSR) of degree 93, 84, and 111.
- Initialization:
  - 80-bit key in left-most registers of first FSR
  - 80-bit IV in left-most registers of second FSR
  - Remaining registers set to 0, except for three right-most registers of third FSR
  - run for 4 x 288 clock ticks to finish init.

Trivium-Initialization

$K_1, \ldots, K_{80}$ Random
$IV_1, \ldots, IV_{80}$ Random
$(a_1, \ldots, a_{93}) \leftarrow (K_1, \ldots, K_{80}, 0, \ldots, 0)$
$(b_1, \ldots, s_{84}) \leftarrow (IV_1, \ldots, IV_{80}, 0, \ldots, 0)$
$(c_1, \ldots, s_{111}) \leftarrow (0, \ldots, 0, 1, 1, 1)$
For $i = 1$ to 4×288 do

1. $t_1 \leftarrow a_{86} + a_{91}a_{92} + b_{79}$
2. $t_2 \leftarrow b_{70} + b_{83}b_{84} + c_1 + c_{87}$
3. $t_3 \leftarrow c_{66} + c_{100}c_{110} + c_{111} + a_{69}$
4. $(a_1, \ldots, a_{93}) \leftarrow (t_3, a_1, \ldots, a_{91})$
5. $(b_1, \ldots, b_{83}) \leftarrow (t_1, b_1, \ldots, b_{82})$
6. $(c_1, \ldots, s_{111}) \leftarrow (t_2, c_1, \ldots, c_{110})$

Note no random bits output. This is just initialization.

Trivium-Iteration

We omit superscripts for readability.
For $i = 1$ to N do

1. $y_i = a_{66} + a_{93} + b_{70} + b_{75} + c_{66} + c_{111}$ (ith random bit).
2. $t_1 \leftarrow a_{86} + a_{91}a_{92} + b_{79}$
3. $t_2 \leftarrow b_{70} + b_{83}b_{84} + c_1 + c_{87}$
4. $t_3 \leftarrow c_{66} + c_{100}c_{110} + c_{111} + a_{69}$
5. $(a_1, \ldots, a_{93}) \leftarrow (t_3, a_1, \ldots, a_{92})$
6. $(b_1, \ldots, b_{83}) \leftarrow (t_1, b_1, \ldots, s_{83})$
7. $(c_1, \ldots, c_{111}) \leftarrow (t_2, c_1, \ldots, c_{110})$

Note the three diff parts of s are three coupled nonlinear FSR.

Trivium based on LFSR though not LFSR

Note:

1. t_1, t_2, t_3 are nonlinear combos of prior bits.
2. $(a_1, \ldots, a_{93}) \leftarrow (t_3, a_1, \ldots, a_{92})$
3. $(b_1, \ldots, b_{83}) \leftarrow (t_1, s_1 \ldots, s_{82})$
4. $(c_1, \ldots, s_{111}) \leftarrow (t_2, s_1, \ldots, s_{110})$

Since t_1, t_2, t_3 nonlinear, Trivium is NOT LFSR
But

Shift to the right and left most bit is BLAH

is very much like LFSR.

Benefit: Shifting is Fast!

Facts About Trivium

1) Has been build in hardware with 3488 logic gates. Small! Fast!
2) So far has not been broken. That we know of!
3) Naive method is 2^{80} steps. Guess all keys.
4) If only do ~ 700 init steps then Cube Attack is 2^{68} steps.
5) Seems to have long period but hard to know:
   1. Nonlin makes it hard to predict. Good for practical A and B.
   2. Nonlin makes it hard to analyze. Bad for theorists A and B.
6) Trivium is also the name of a rock band!
7) Two Papers on Trivium on course website

Why the name Trivium?

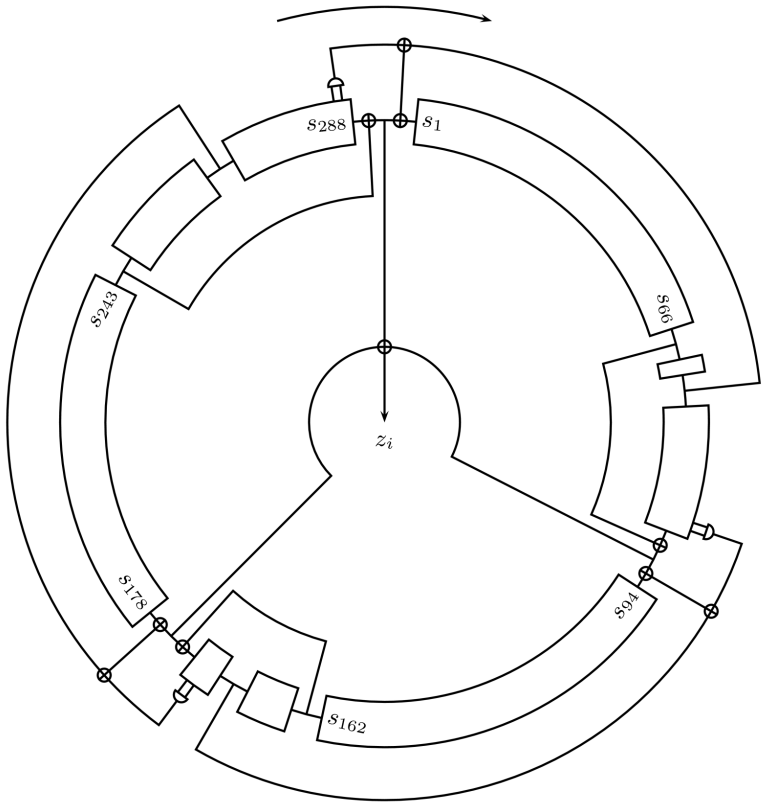
We quote the paper

> *The word trivium is Latin for "the three-fold way", and refers to the three-fold symmetry of TRIVIUM. The adjective trivial which was derived from it, has a connotation of simplicity, which is also one of the characteristics of TRIVIUM.*

(Quote continued on next slide)

Why the name Trivium?

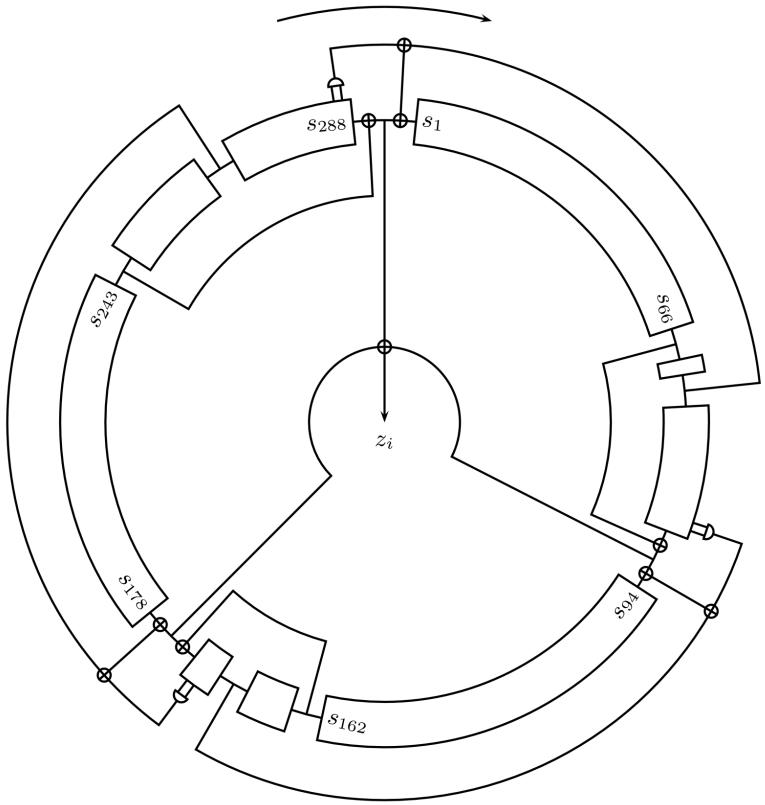
Moreover, with some imagination, one might recognize the shape of a Trivial Pursuit board in Fig. 1



(Quote continued on next slide)

Why the name Trivium?

While we admit this respect "Mercedes" would have been a more appropriate name.



Why the name Trivium?

Finally, the name provides a nice title for a subsequence cryptanalysis paper: "Three Trivial Attacks on Trivium".