

Defeating the German Enigma

CRYPTOLOGY

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The study of making and breaking ciphers

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- Cryptography: The study of making ciphers.

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- Cryptanalysis: The study of breaking ciphers.

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- What is the size of the key space? 26.
- How hard is it to guess?

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MFXCSYGERVIEHLMW

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MFIXCSYGERVIEHXLW

NGJYDTZHFSWJFIYMN

LEHWBRXFDQUHDGWKLV

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MFXCSYGERVIEHXMLW
NGJYDTZHFSWJFIYMNX
OHKZEUAIGTXKGJZNOY

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MFXCSYGERVIEHXMLW
NGJYDTZHFSWJFIYMNX
OHKZEUAIGTXKGJZNOY
PILAFVBJHUYLHKAOPZ
QJMBGWCKIVZMILBPQA
RKNCHXDLJWANJMCQRB
SLODIYEMKXBOKNDRSC
TMPEJZFNLYCPLOESTD
UNQFKAGOMZDQMPFTUE
VORGLBHPNAERNQGUVF
WPSHMCIQOBFSORHVWG
XQTINDJRPCGTPSIWXH

YRUJOEKSQDHUQTJXYI
ZSVKPFLTREIVRUKYZJ
ATWLQGMUSFJWSVLZAK
BUXMRHNVTGKXTWMABL
CVYNSIOWUHLYUXNBCM
DWZOTJPXVIMZVYOCDN
EXAPUKQYWJNAWZPDEO
FYBQVLRZKKOBXAQEFP
GZCRWMSAYLPCYBRFGQ
HADSXNTBZMQDZCSGHR
IBETYOUCANREADTHIS
JCFUZPVDBOSFBEUIJT
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MFXCSYGERVIEHXMLW
NGJYDTZHFSWJFIYMNX
OHKZEUAIGTXKGJZNOY
PILAFVBJHUYLHKAOPZ
QJMBGWCKIVZMILBPQA
RKNCHXDLJWANJMCQRB
SLODIYEMKXBOKNDRSC
TMPEJZFNLYCPLOESTD
UNQFKAGOMZDQMPFTUE
VORGLBHPNAERNQGUVF
WPSHMCIQOBFSORHVWG
XQTINDJRPCGTPSIWXH

YRUJOEKSQDHUQTJXYI
ZSVKPFLTREIVRUKYZJ
ATWLQGMUSFJWSVLZAK
BUXMRHNVTGKXTWMABL
CVYNSIOWUHLYUXNBCM
DWZOTJPXVIMZVYOCDN
EXAPUKQYWJNAWZPDEO
FYBQVLRZKKOBXAQEFP
GZCRWMSAYLPCYBRFGQ
HADSXNTBZMQDZCSGHR
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JCFUZPVDBOSFBEUIJT
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Monoalphabetic Substitution Ciphers

Use a full permutation of the alphabet.

Plaintext: ABCDEFGHIJKLMNOPQRSTUVWXYZ

Ciphertext: krusfltagzmqnbveophxjywcid

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- Monoalphs remained secure for centuries. Arab cryptanalysts discovered *frequency analysis* around 800 A.D.

Frequency of Letters in English

Letter	Percentage		Letter	Percentage
A	8.2		N	6.7
B	1.5		O	7.5
C	2.8		P	1.9
D	4.3		Q	0.1
E	12.7		R	6.0
F	2.2		S	6.3
G	2.0		T	9.1
H	6.1		U	2.8
I	7.0		V	1.0
J	0.2		W	2.4
K	0.8		X	0.2
L	4.0		Y	2.0
M	2.4		Z	0.1

- (Source: H. Beker and F. Piper, *Cipher Systems: The Protection of Communication*.)

Example of Frequency Analysis and Language Modelling

LXV TWJ RXZGR KL TVSZJGBJ WJTBEZXG EX EIZN Z PJTG EIZN ZN TG

TVSZJGBJ EITE ZN WTZNJS XG EJYJMZNZXG EIJZW NETGSTWSN ITMJ KJJG

YXDJWJS XMJW EIJ LJTNW LXV HGXD EIJNJ RVLN NZE ZG AWXGE XA EIJZW

NJEN TGS EIJ RTPPT WTLN JTE EIJ DIZEJ KYXXS BJYYN XA EIJZW

KWTZGN XVE VI LXV HGXD Z FVZE

A:0.013

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E:0.098

F:0.004

G:0.071

H:0.008

I:0.058

J:0.129

K:0.017

L:0.031

M:0.013

N:0.075

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YXDeWeS XMeW Eie LeTWN LXV HGXD EieNe RVLN NZE ZG AWXGE XA EieZW

NeEN TGS Eie RTPPT WTLN eTE Eie DIZEe KYXXS BeYYN XA EieZW

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YXDeWeS XMeW **E**Ie LeTWN LXV HGXD **E**IeNe RVLN **NZE** ZG AWX**E** XA **E**IeZW

Ne**EN** TGS **E**Ie RTPPT WTLN e**TE** **E**Ie **DIZE**e KYXXS BeYYN XA **E**IeZW

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M:0.013	N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	S:0.035	a-T:0.080
U:0.000	V:0.040	r-W:0.058	o-X:0.080
Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

LoV are RoiGR KL aVSieGBe reaBtioG to this i PeaG this is aG

aVSieGBe that is raises oG teYeMisioG their staGSarSs haMe KeeG

YoDereS oMer the Lears LoV HGoD these RVLs sit iG AroGt oA their

sets aGS the RaPPa raLs eat the Dhite KYooS BeYYs oA their

KraiGs oVt Vh LoV HGoD i FVit

A:0.013	B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	L:0.031
M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	S:0.035	a-T:0.080
U:0.000	V:0.040	r-W:0.058	o-X:0.080
Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

LoV are RoiGR KL aVSiGBe reaBtioG to this i PeaG this is aG

aVSiGBe that is raises oG teYeMisioG their staGSarSs haMe KeeG

YoDereS oMer the Lears LoV HGoD these RVLs sit iG AroGt oA their

sets aGS the RaPPa raLs eat the Dhite KYooS BeYYs oA their

KraiGs oVt Vh LoV HGoD i FVit

A:0.013	B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	L:0.031
M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	S:0.035	a-T:0.080
U:0.000	V:0.040	r-W:0.058	o-X:0.080
Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

Lou are RoiGR KL auSieGBe reaBtioG to this i PeaG this is aG
auSieGBe that is raises oG teYeMisioG their staGSarSs haMe KeeG
YoDereS oMer the Lears Lou HGoD these RuLs sit iG AroGt oA their
sets aGS the RaPPa raLs eat the Dhite KYooS BeYYs oA their
KraiGs out uh Lou HGoD i Fuit

A:0.013	B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	L:0.031
M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

Lou are RoiGR KL auSieGBe reaBtioG to this i PeaG this is aG
auSieGBe that is raises oG teYeMisioG their staGSarSs haMe KeeG
YoDereS oMer the Lears Lou HGoD these RuLs sit iG AroGt oA their
sets aGS the RaPPa raLs eat the Dhite KYooS BeYYs oA their
KraiGs out uh Lou HGoD i Fuit

A:0.013	B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	L:0.031
M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are RoiGR Ky auSieGBe reaBtioG to this i PeaG this is aG

auSieGBe that is raises oG teYeMisioG their staGSarSs haMe KeeG

YoDereS oMer the years you HGoD these Ruys sit iG AroGt oA their

sets aGS the RaPPa rays eat the Dhite KYooS BeYYs oA their

KraiGs out uh you HGoD i Fuit

A:0.013	B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are RoiGR Ky auSieGBe reaBtioG to this i PeaG this is aG

auSieGBe that is raises oG teYeMisioG their staGSarSs haMe KeeG

YoDereS oMer the years you HGoD these Ruys sit iG AroGt oA their

sets aGS the RaPPa rays eat the Dhite KYooS BeYYs oA their

KraiGs out uh you HGoD i Fuit

A:0.013	B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are RoinR Ky auSienBe reaBtion to this i Pean this is an
auSienBe that is raisesS on teYeMision their stanSarSs haMe Keen
YoDereS oMer the years you HnoD these Ruys sit in Aront oA their
sets anS the RaPPa rays eat the Dhite KYooS BeYYs oA their
Krains out uh you HnoD i Fuit

A:0.013	B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are RoinR Ky auSienBe reaBtion to this i Pean this is an
auSienBe that is raises on teYeMision their stanSarSs haMe Keen
YoDereS oMer the years you HnoD these Ruys sit in Aront oA their
sets anS the RaPPa rays eat the Dhite KYooS BeYYs oA their
Krains out uh you HnoD i Fuit

A:0.013	B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are RoinR Ky auScience reaction to this i Pean this is an
auScience that is raisesS on teYeMision their stanSarSs haMe Keen
YoDereS oMer the years you HnoD these Ruys sit in Aront oA their
sets anS the RaPPa rays eat the Dhite KYooS ceYYs oA their
Krains out uh you HnoD i Fuit

A:0.013	c-B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are RoinR Ky auScience reaction to this i Pean this is an
auScience that is raiseS on teYeMision their stanSarSs haMe Keen
YoDereS oMer the years you HnoD these Ruys sit in Aront oA their
sets anS the RaPPa rays eat the Dhite KYooS ceYYs oA their
Krains out uh you HnoD i Fuit

A:0.013	c-B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are RoinR Ky audience reaction to this i Pean this is an
audience that is raised on teYeMision their standards haMe Keen
YoDered oMer the years you HnoD these Ruys sit in Aront oA their
sets and the RaPPa rays eat the Dhite KYood ceYYs oA their
Krains out uh you HnoD i Fuit

A:0.013	c-B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	d-S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
Y:0.022	i-Z:0.093		

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you are RoinR Ky audience reaction to this i Pean this is an
audience that is raised on teYeMision their standards haMe Keen
YoDered oMer the years you HnoD these Ruys sit in Aront oA their
sets and the RaPPa rays eat the Dhite KYood ceYYs oA their
Krains out uh you HnoD i Fuit

A:0.013	c-B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	d-S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are RoinR Ky audience reaction to this i Pean this is an
audience that is raised on teleMision their standards haMe Keen
loDered oMer the years you HnoD these Ruys sit in Aront oA their
sets and the RaPPa rays eat the Dhite Kllood cells oA their
Krains out uh you HnoD i Fuit

A:0.013	c-B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	d-S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
l-Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are RoinR Ky audience reaction to this i Pean this is an
audience that is raised on teleMision their standards haMe Keen
loDered oMer the years you HnoD these Ruys sit in Aront oA their
sets and the RaPPa rays eat the Dhite Kllood cells oA their
Krains out uh you HnoD i Fuit

A:0.013	c-B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	d-S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
l-Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are RoinR Ky audience reaction to this i Pean this is an
audience that is raised on television their standards have Keen
loDered over the years you HnoD these Ruys sit in Aront oA their
sets and the RaPPa rays eat the Dhite Kllood cells oA their
Krains out uh you HnoD i Fuit

A:0.013	c-B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
v-M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	d-S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
l-Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are RoinR Ky audience reaction to this i Pean this is an
audience that is raised on television their standards have Keen
loDered over the years you HnoD these Ruys sit in Aront oA their
sets and the RaPPa rays eat the Dhite Kllood cells oA their
Krains out uh you HnoD i Fuit

A:0.013	c-B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
v-M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	d-S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
l-Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are RoinR Ky audience reaction to this i Pean this is an
audience that is raised on television their standards have Keen
loDered over the years you HnoD these Ruys sit in front of their
sets and the RaPPa rays eat the Dhite Kllood cells of their
Krains out uh you HnoD i Fuit

f-A:0.013	c-B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
v-M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	d-S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
l-Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are **RoinR** Ky audience reaction to this i Pean this is an
audience that is raised on television their standards have Keen
loDered over the years you HnoD these **Ruys** sit in front of their
sets and the **RaPPa** rays eat the Dhite Kllood cells of their
Krains out uh you HnoD i Fuit

f-A:0.013	c-B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
v-M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	R:0.017	d-S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
l-Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are going Ky audience reaction to this i Pean this is an
audience that is raised on television their standards have Keen
loDered over the years you HnoD these guys sit in front of their
sets and the gaPPa rays eat the Dhite Kllood cells of their
Krains out uh you HnoD i Fuit

f-A:0.013	c-B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
v-M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	g-R:0.017	d-S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
l-Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are going Ky audience reaction to this i Pean this is an
audience that is raised on television their standards have Keen
loDered over the years you HnoD these guys sit in front of their
sets and the gaPPa rays eat the Dhite Kllood cells of their
Krains out uh you HnoD i Fuit

f-A:0.013	c-B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
v-M:0.013	s-N:0.075	O:0.000	P:0.013
Q:0.000	g-R:0.017	d-S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
l-Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are going Ky audience reaction to this i mean this is an
audience that is raised on television their standards have Keen
loDered over the years you HnoD these guys sit in front of their
sets and the gamma rays eat the Dhite Kllood cells of their
Krains out uh you HnoD i Fuit

f-A:0.013	c-B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
v-M:0.013	s-N:0.075	O:0.000	m-P:0.013
Q:0.000	g-R:0.017	d-S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
l-Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are going **K**y audience reaction to this i mean this is an
audience that is raised on television their standards have **K**een
loDered over the years you HnoD these guys sit in front of their
sets and the gamma rays eat the Dhite **K**lood cells of their
Krain's out uh you HnoD i Fuit

f-A:0.013	c-B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	K:0.017	y-L:0.031
v-M:0.013	s-N:0.075	O:0.000	m-P:0.013
Q:0.000	g-R:0.017	d-S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
l-Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are going by audience reaction to this i mean this is an
audience that is raised on television their standards have been
lowered over the years you know these guys sit in front of their
sets and the gamma rays eat the white blood cells of their
brains out uh you know i find

f-A:0.013	c-B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	b-K:0.017	y-L:0.031
v-M:0.013	s-N:0.075	O:0.000	m-P:0.013
Q:0.000	g-R:0.017	d-S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
l-Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are going by audience reaction to this i mean this is an
audience that is raised on television their standards have been
lowered over the years you know these guys sit in front of their
sets and the gamma rays eat the white blood cells of their
brains out uh you know i find

f-A:0.013	c-B:0.017	C:0.000	D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	b-K:0.017	y-L:0.031
v-M:0.013	s-N:0.075	O:0.000	m-P:0.013
Q:0.000	g-R:0.017	d-S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
l-Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are going by audience reaction to this i mean this is an
audience that is raised on television their standards have been
lowered over the years you Hnow these guys sit in front of their
sets and the gamma rays eat the white blood cells of their
brains out uh you Hnow i Fuit

f-A:0.013	c-B:0.017	C:0.000	w-D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	b-K:0.017	y-L:0.031
v-M:0.013	s-N:0.075	O:0.000	m-P:0.013
Q:0.000	g-R:0.017	d-S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
l-Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are going by audience reaction to this i mean this is an
audience that is raised on television their standards have been
lowered over the years you **H**now these guys sit in front of their
sets and the gamma rays eat the white blood cells of their
brains out uh you **H**now i Fuit

f-A:0.013	c-B:0.017	C:0.000	w-D:0.017
t-E:0.098	F:0.004	n-G:0.071	H:0.008
h-I:0.058	e-J:0.129	b-K:0.017	y-L:0.031
v-M:0.013	s-N:0.075	O:0.000	m-P:0.013
Q:0.000	g-R:0.017	d-S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
l-Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are going by audience reaction to this i mean this is an
audience that is raised on television their standards have been
lowered over the years you know these guys sit in front of their
sets and the gamma rays eat the white blood cells of their
brains out uh you know i Fuit

f-A:0.013	c-B:0.017	C:0.000	w-D:0.017
t-E:0.098	F:0.004	n-G:0.071	k-H:0.008
h-I:0.058	e-J:0.129	b-K:0.017	y-L:0.031
v-M:0.013	s-N:0.075	O:0.000	m-P:0.013
Q:0.000	g-R:0.017	d-S:0.035	a-T:0.080
U:0.000	u-V:0.040	r-W:0.058	o-X:0.080
l-Y:0.022	i-Z:0.093		

Example of Frequency Analysis and Language Modelling

you are going by audience reaction to this i mean this is an
audience that is raised on television their standards have been
lowered over the years you know these guys sit in front of their
sets and the gamma rays eat the white blood cells of their
brains out uh you know i **F**uit

f-A:0.013	c-B:0.017	C:0.000	w-D:0.017
t-E:0.098	F:0.004	n-G:0.071	k-H:0.008
h-I:0.058	e-J:0.129	b-K:0.017	y-L:0.031
v-M:0.013	s-N:0.075	O:0.000	m-P:0.013
Q:0.000	g-R:0.017	d-S:0.035	a-T:0.080
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Polyalphabetic Ciphers

- Vigenère cipher: Use a keyword to interleave several Caesar shifts.

```
homerhomerhomerho  
WELCOMETOILLINOIS  
dsxgftsfszszurfpq
```

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MNOMTTITUARMQSQLOZBEXMVUAI AVTTMIDATMVDMZDEPAHMBQMNXW
WQZEPWVQZTTMYQIREGOGSNAETTMSQOUKASUBIZNRAVTANTTMIDAE
FAAZLTTMGMUMMZAKAEMBTTMWTQTQJLAWDOMLXAORBHQQRNZAUVSA
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FAAZLTTMGMUMMZAKAEMBTTMWTQTQJLAWDOMLXAORBHQQRNZAUVSA
CTGPYACKZWWUYUUB

- Look how the frequency distribution is flattened out:

A:0.0577	B:0.0222	C:0.0088	D:0.0355	E:0.0888
F:0.0622	G:0.0222	H:0.0222	I:0.0711	J:0.0044
K:0.0355	L:0.0044	M:0.0711	N:0.0266	O:0.0266
P:0.0577	Q:0.0622	R:0.0222	S:0.0488	T:0.0577
U:0.0577	V:0.0266	W:0.0488	X:0.0088	Y:0.0088
Z:0.0400				

Vigenère Cipher (Cont.)

- The Vigenère cipher remained unbroken for about three hundred years. Babbage discovered the following weakness:

KWUMZESWIZOBKIUPQEZKEDMAOBIAVTABHUA IYMAZBHUAIEINMCDU
MNOMTTITUARMQSQLOZBEXMVUAI AVTTMIDATMVDMZDEPAHMBQMNXW
WQZEPWVQZTTMYQIREGOGSNAETTMSQOUKASUBIZNRAVTANTTMIDAE
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WQZEPWVQZTTMYQIREGOGSNAE **TTM**SQOUKASUBIZNRAVTAN **TTM**IDAE
FAAZL **TTM**GMUMMZAKAEMB **TTM**WTQTQJLAWDOMLXAORBHQQRNZAUVSA
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- Common letter blocks spaced apart by a multiple of the keyword will encrypt identically.

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MNOMTTITUARMQSQLOZBEXMVUAI AVTTMIDATMVDMZDEPAHMBQMNXW
WQZEPWVQZTTMYQIREGOGSNAET<----- 21 ----->TTMIDAE
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MNOMTTITUARMQSQLOZBEXMVUAI AVTTMIDATMVDMZDEPAHMBQMNXW
WQZEPWVQZTTMYQIREGOGSNAETTMSQOUKASUBIZNRAVTANTTMIDAE
FAAZL**T**<---- 15 ---->**TTM**WTQTQJLAWDOMLXAORBHQQRNZAUVSA
CTGPYACKZWWUYUUB

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- Common letter blocks spaced apart by a multiple of the keyword will encrypt identically.
- Key length divides GCD of identical spacings. Here $\text{gcd}(15, 21) = 3$.
- Once key length is discovered, perform multiple frequency counts.

A:0.1200
B:0.0000
C:0.0000
D:0.0400
E:0.0400
F:0.0133
G:0.0267
H:0.0133
I:0.0000
J:0.0000
K:0.0533
L:0.0000
M:0.1067
N:0.0133
O:0.0400
P:0.0267
Q:0.1067
R:0.0133
S:0.0133
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F:0.0000
G:0.0133
H:0.0000
I:0.0533
J:0.0133
K:0.0133
L:0.0267
M:0.1867
N:0.0267
O:0.0267
P:0.0267
Q:0.0533
R:0.0000
S:0.0133
T:0.0000
U:0.0133
V:0.0667
W:0.0800
X:0.0000
Y:0.0133
Z:0.0800

A:0.0811
B:0.0270
C:0.0000
D:0.0541
E:0.0946
F:0.0000
G:0.0135
H:0.0405
I:0.1081
J:0.0000
K:0.0135
L:0.0270
M:0.0135
N:0.0541
O:0.0405
P:0.0000
Q:0.0000
R:0.0541
S:0.0541
T:0.1757
U:0.0541
V:0.0270
W:0.0405
X:0.0000
Y:0.0270
Z:0.0000

o-A:0.1200	A:0.1333	A:0.0811
p-B:0.0000	B:0.1067	B:0.0270
q-C:0.0000	C:0.0400	C:0.0000
r-D:0.0400	D:0.0000	D:0.0541
s-E:0.0400	E:0.0133	E:0.0946
t-F:0.0133	F:0.0000	F:0.0000
u-G:0.0267	G:0.0133	G:0.0135
v-H:0.0133	H:0.0000	H:0.0405
w-I:0.0000	I:0.0533	I:0.1081
x-J:0.0000	J:0.0133	J:0.0000
y-K:0.0533	K:0.0133	K:0.0135
z-L:0.0000	L:0.0267	L:0.0270
a-M:0.1067	M:0.1867	M:0.0135
b-N:0.0133	N:0.0267	N:0.0541
c-O:0.0400	O:0.0267	O:0.0405
d-P:0.0267	P:0.0267	P:0.0000
e-Q:0.1067	Q:0.0533	Q:0.0000
f-R:0.0133	R:0.0000	R:0.0541
g-S:0.0133	S:0.0133	S:0.0541
h-T:0.1067	T:0.0000	T:0.1757
i-U:0.1200	U:0.0133	U:0.0541
j-V:0.0000	V:0.0667	V:0.0270
k-W:0.0000	W:0.0800	W:0.0405
l-X:0.0400	X:0.0000	X:0.0000
m-Y:0.0133	Y:0.0133	Y:0.0270
n-Z:0.0933	Z:0.0800	Z:0.0000

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Vigenère Cipher (Cont.)

- Keyword = MIA

youaregoingbyaudiencereactiontothisimeanthisisanaudi
KWUMZESWIZOBKIUPQEZKEDMAOBIAVTABHUA IYMAZBHUAIEINMCDU

encethatisraisedontelevisiontheirstandardshavebeenlo
MNOMTTITUARMQSQLOZBEXMVUAI AVTTMIDATMVDMZDEPAHMBQMNXW

weredovertheyearsyouknowtheseguyssitinfrontoftheirse
WQZEPWVQZTTMYQIREGOGSNAETTMSQOUKASUBIZNRAVTANTTMIDAE

tsandthegammarayseatthewhitebloodcellsofthierbrainso
FAAZLTTMGMUMMZAKAEMBTTMWTQTQJLAWDOMLXAORBHQQRNZAUVSA

utuhyouknowiquit
CTGPYACKZWWUYUUB

Vigenère Cipher (Cont.)

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u e i b u e e a i t h i a h i n d
U E I B U E E A I T H I A H I N D

n t t r s o e v i t i t d d a b n
N T T R S O E V I T I T D D A B N

w e v t y r o n t s u s i r t t i e
W E V T Y R O N T S U S I R T T I E

a t g m a e t w t l d l o h r a s
A T G M A E T W T L D L O H R A S

t y k w u
T Y K W U

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Perfecting the Vigenère Cipher

- What if the keyword in a Vigenère cipher is chosen to be long enough that it never cycles?

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ONE-TIME PADS

Vigenère ciphers whose keyword is as long as the message itself.

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guessed keyword: REAZPKHRKCZK
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```
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cipher  : FLNNNYBUSFMD
guessed keyword: REAZPKHRKCZK
incorrect decrypt: OHNOYOU DIDNT
```

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XTCTSBHZWLCSCCTGSSNTOWUKYDFXPVT

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- This is vulnerable to *cribbing*.

One-time Pads (Cont.)

Guess the location of “the” .

XTCTSBHZWLCSTGSSNTOWUKYDFXPVT

QHNNFIJNWSSOTGUJRNOXFMHRESMHVP

OERZTOHXANSASLKWELZBAOJVUSOUOV

One-time Pads (Cont.)

Guess the location of “the” .

XTCTSBHZWLCSTGSSNTOWUKYDFXPVT

crib--> the

QHNNFIJNWSSOTGUJRNOXFMHRESMHVP

key --> xaj

OERZTOHXANSASLKWELZBAOJVUSOUOV

One-time Pads (Cont.)

Find corresponding decrypts. Guess again.

att

XTCTSBHZWLCSTGSSNTOWUKYDFXPVT

xaj

crib--> the

QHNNFIJNWSSOTGUJRNOXFMHRESMHVP

key --> xaj

rei

OERZTOHXANSASLKWELZBAOJVUSOUOV

xaj

One-time Pads (Cont.)

Find corresponding decrypts.

crib--> attack

XTCTSBHZWLCSTGSSNTOWUKYDFXPVT

key --> xajtqr

the

QHNNFIJNWSSOTGUJRNOXFMHRESMHVP

xaj

rei

OERZTOHXANSASLKWELZBAOJVUSOUOV

xaj

One-time Pads (Cont.)

A wrong assumption.

attack

XTCTSBHZWLCSTGSSNTOWUKYDFXPVT

xajtqr

theupr

QHNNFIJNWSSOTGUJRNOXFMHRESMHVP

xajtqr

reigdx

OERZTOHXANSASLKWELZBAOJVUSOUOV

xajtqr

One-time Pads (Cont.)

Back up.

att

XTCTSBHZWLCSTGSSNTOWUKYDFXPVT

xaj

the

QHNNFIJNWSSOTGUJRNOXFMHRESMHVP

xaj

rei

OERZTOHXANSASLKWELZBAOJVUSOUOV

xaj

One-time Pads (Cont.)

Crib again. Looks good!

crib--> atthe

XTCTSBHZWLCSTGSSNTOWUKYDFXPVT

key --> xajmo

thebr

QHNNFIJNWSSOTGUJRNOXFMHRESMHVP

xajmo

reinf

OERZTOHXANSASLKWELZBAOJVUSOUOV

xajmo

One-time Pads (Cont.)

Crib again.

atthebrea

XTCTSBHZWLCSTGSSNTOWUKYDFXPVT

xajmoaqvw

thebritsa

QHNNFIJNWSSOTGUJRNOXFMHRESMHVP

xajmoaqvw

crib--> reinforce

OERZTOHXANSASLKWELZBAOJVUSOUOV

xajmoaqvw

One-time Pads (Cont.)

... which leads to ...

crib--> atthebreak

XTCTSBHZWLCSTGSSNTOWUKYDFXPVT

xajmoaqvwb

thebritsar

QHNNFIJNWSSOTGUJRNOXFMHRESMHVP

xajmoaqvwb

reinforcem

OERZTOHXANSASLKWELZBAOJVUSOUOV

xajmoaqvwb

One-time Pads (Cont.)

... which leads to ...

atthebreakofda

XTCTSBHZWLCSTGSSNTOWUKYDFXPVT

xajmoaqvwbonzt

thebritsarebun

QHNNFIJNWSSOTGUJRNOXFMHRESMHVP

xajmoaqvwbonzt

crib--> reinforcements

OERZTOHXANSASLKWELZBAOJVUSOUOV

xajmoaqvwbonzt

One-time Pads (Cont.)

... which leads to ... ??? Back up and try again.

crib--> atthebreakofday

XTCTSBHZWLCSTGSSNTOWUKYDFXPVT

xajmoaqvwbonzti

thebritsarebunm

QHNNFIJNWSSOTGUJRNOXFMHRESMHVP

xajmoaqvwbonzti

reinforcementsc

OERZTOHXANSASLKWELZBAOJVUSOUOV

xajmoaqvwbonzti

One-time Pads (Cont.)

Much better! Keep going ...

crib--> atthebreakofdawn

XTCTSBHZWLCSTGSSNTOWUKYDFXPVT

xajmoaqvwbonztkf

thebritsarebunke

QHNNFIJNWSSOTGUJRNOXFMHRESMHVP

xajmoaqvwbonztkf

reinforcementsar

OERZTOHXANSASLKWELZBAOJVUSOUOV

xajmoaqvwbonztkf

One-time Pads (Cont.)

Keep going ...

atthebreakofdawnsei
XTCTSBHZWLCSTGSSNTOWUKYDFXPVT
xajmoaqvwbonztkfajl

crib--> thebritsarebunkered
QHNNFIJNWSSOTGUJRNOXFMHRESMHVP
xajmoaqvwbonztkfajl

reinforcementsareco
OERZTOHXANSASLKWELZBAOJVUSOUOV
xajmoaqvwbonztkfajl

One-time Pads (Cont.)

Almost there ...

crib--> atthebreakofdawnseize
XTCTSBHZWLCSTGSSNTOWUKYDFXPVT
xajmoaqvwbonztkfajlps

thebritsarebunkeredin
QHNNFIJNWSSOTGUJRNOXFMHRESMHVP
xajmoaqvwbonztkfajlps

reinforcementsarecomi
OERZTOHXANSASLKWELZBAOJVUSOUOV
xajmoaqvwbonztkfajlps

One-time Pads (Cont.)

Almost there ...

atthebreakofdawnseizeth
XTCTSBHZWLCSTGSSNTOWUKYDFXPVT
xajmoaqvwbonztkfajlpsbd

thebritsarebunkeredinle
QHNNFIJNWSSOTGUJRNOXFMHRESMHVP
xajmoaqvwbonztkfajlpsbd

crib--> reinforcementsarecoming
OERZTOHXANSASLKWELZBAOJVUSOUOV
xajmoaqvwbonztkfajlpsbd

One-time Pads (Cont.)

Almost there ...

crib--> atthebreakofdawnseizethe
XTCTSBHZWLCSTGSSNTOWUKYDFXPVT
xajmoaqvwbonztkfajlpsbdu

thebritsarebunkeredinlex
QHNNFIJNWSSOTGUJRNOXFMHRESMHVP
xajmoaqvwbonztkfajlpsbdu

reinforcementsarecomingb
OERZTOHXANSASLKWELZBAOJVUSOUOV
xajmoaqvwbonztkfajlpsbdu

One-time Pads (Cont.)

... got it!

atthebreakofdawnseizetheharbor
XTCTSBHZWLCSTGSSNTOWUKYDFXPVT
xajmoaqvwbonztkfajlpsbduwfgohc

crib--> thebritsarebunkeredinlexington
QHNNFIJNWSSOTGUJRNOXFMHRESMHVP
xajmoaqvwbonztkfajlpsbduwfgohc

reinforcementsarecomingbynight
OERZTOHXANSASLKWELZBAOJVUSOUOV
xajmoaqvwbonztkfajlpsbduwfgohc

Background of Enigma

- After the French cracked the ADFGVX cipher in WWI, Arthur Scherbius set out to create an improved cipher machine.

Background of Enigma

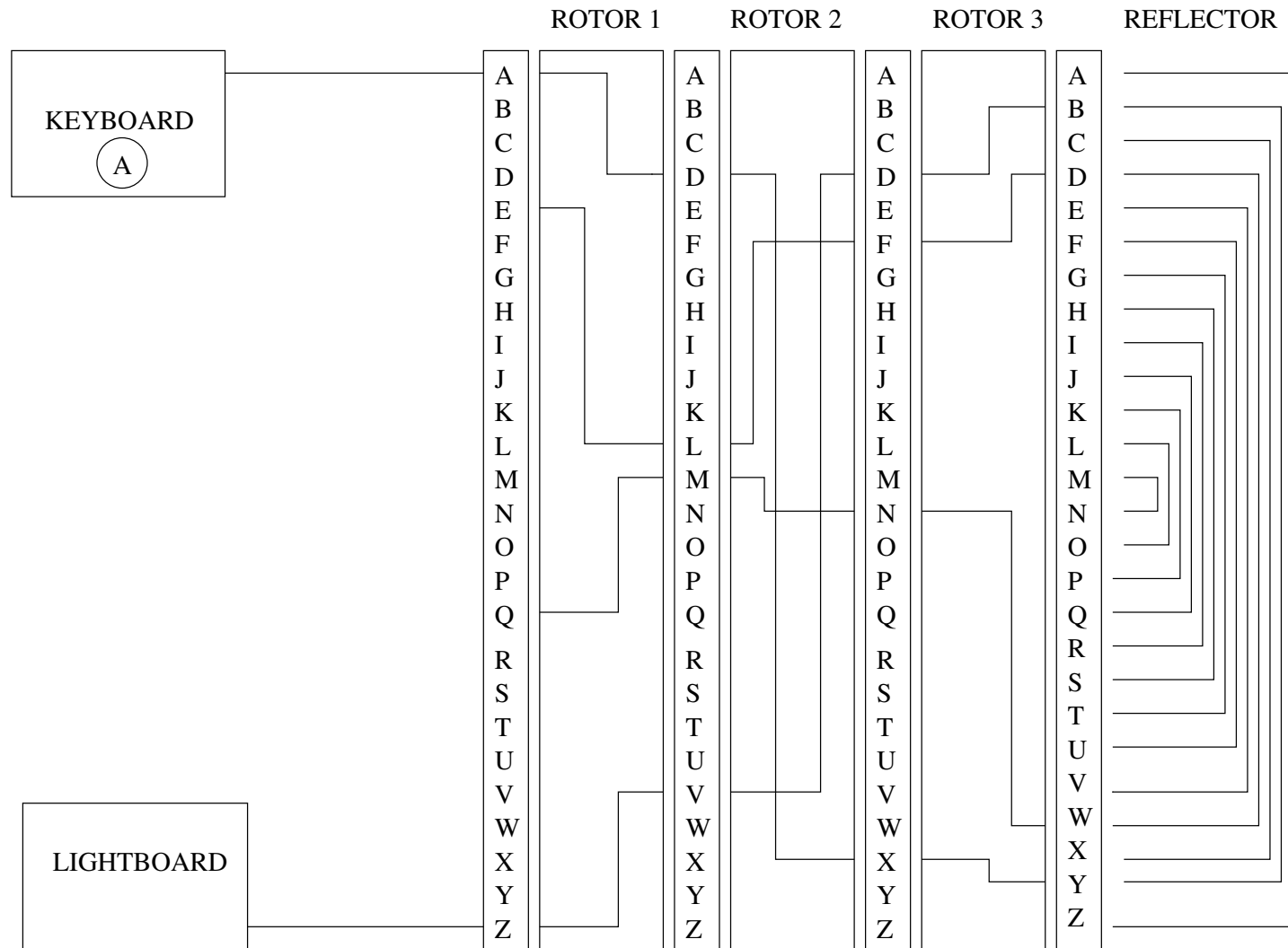
- After the French cracked the ADFGVX cipher in WWI, Arthur Scherbius set out to create an improved cipher machine.
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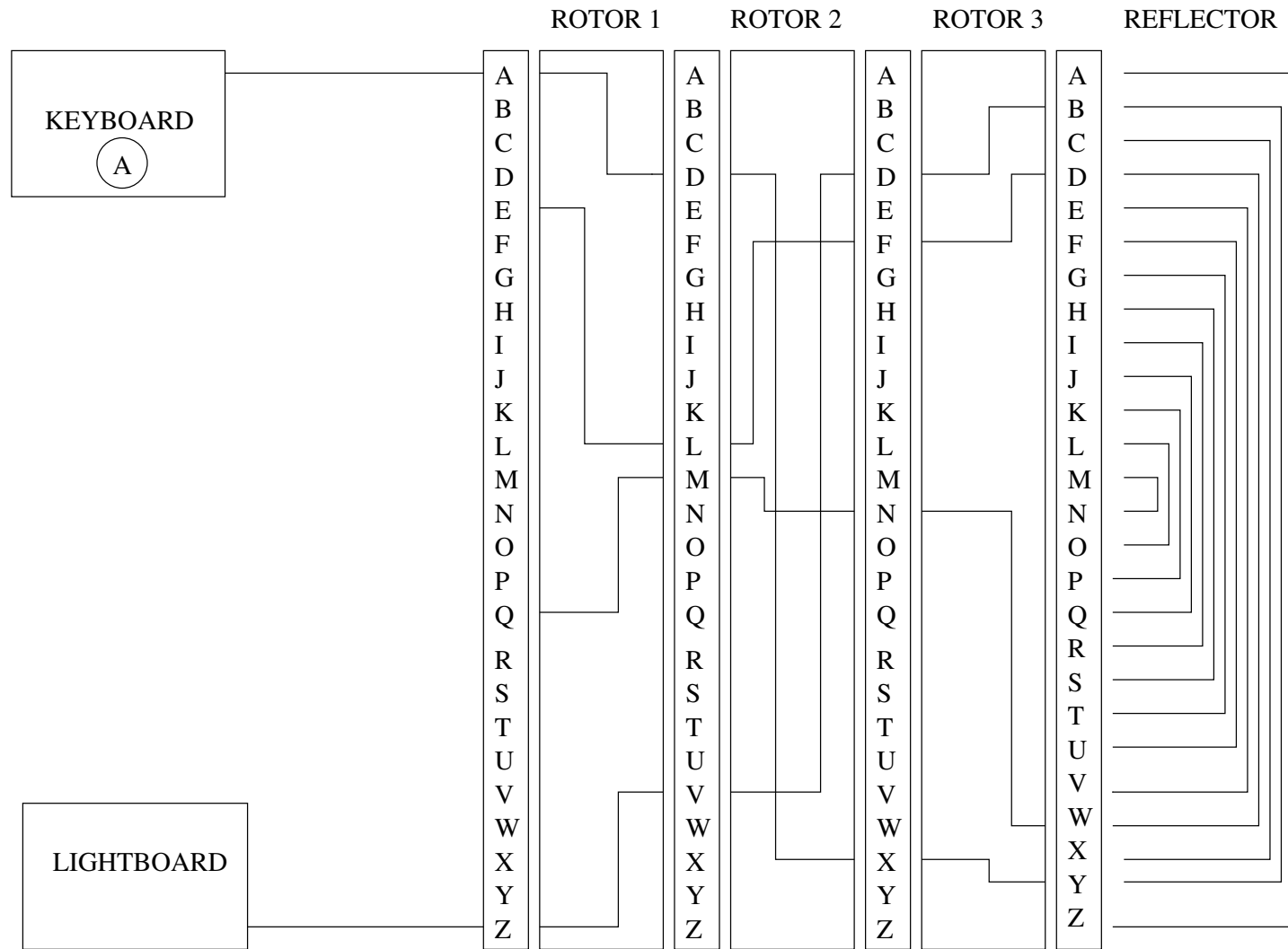
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Enigma Schematic

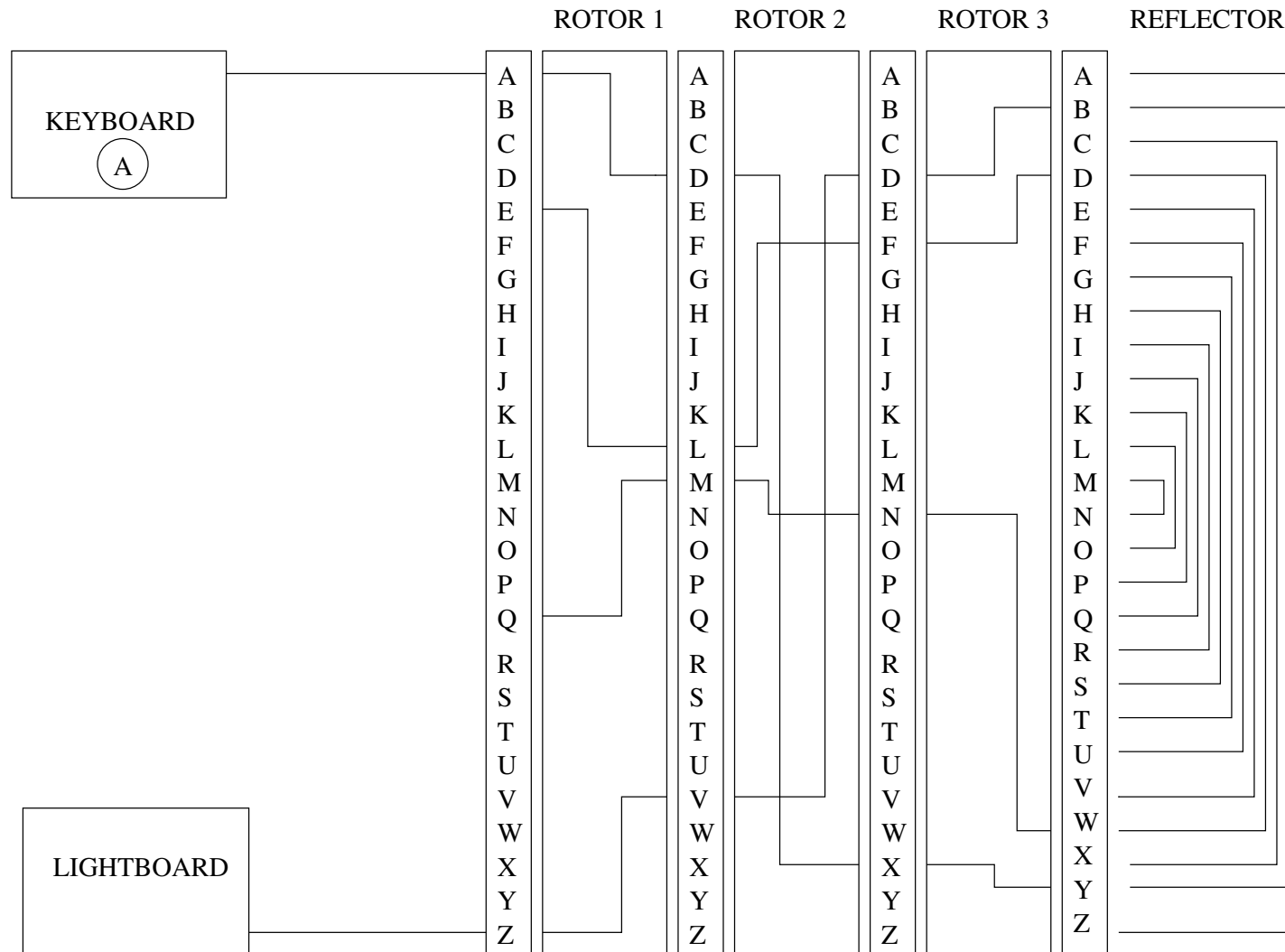


Enigma Schematic



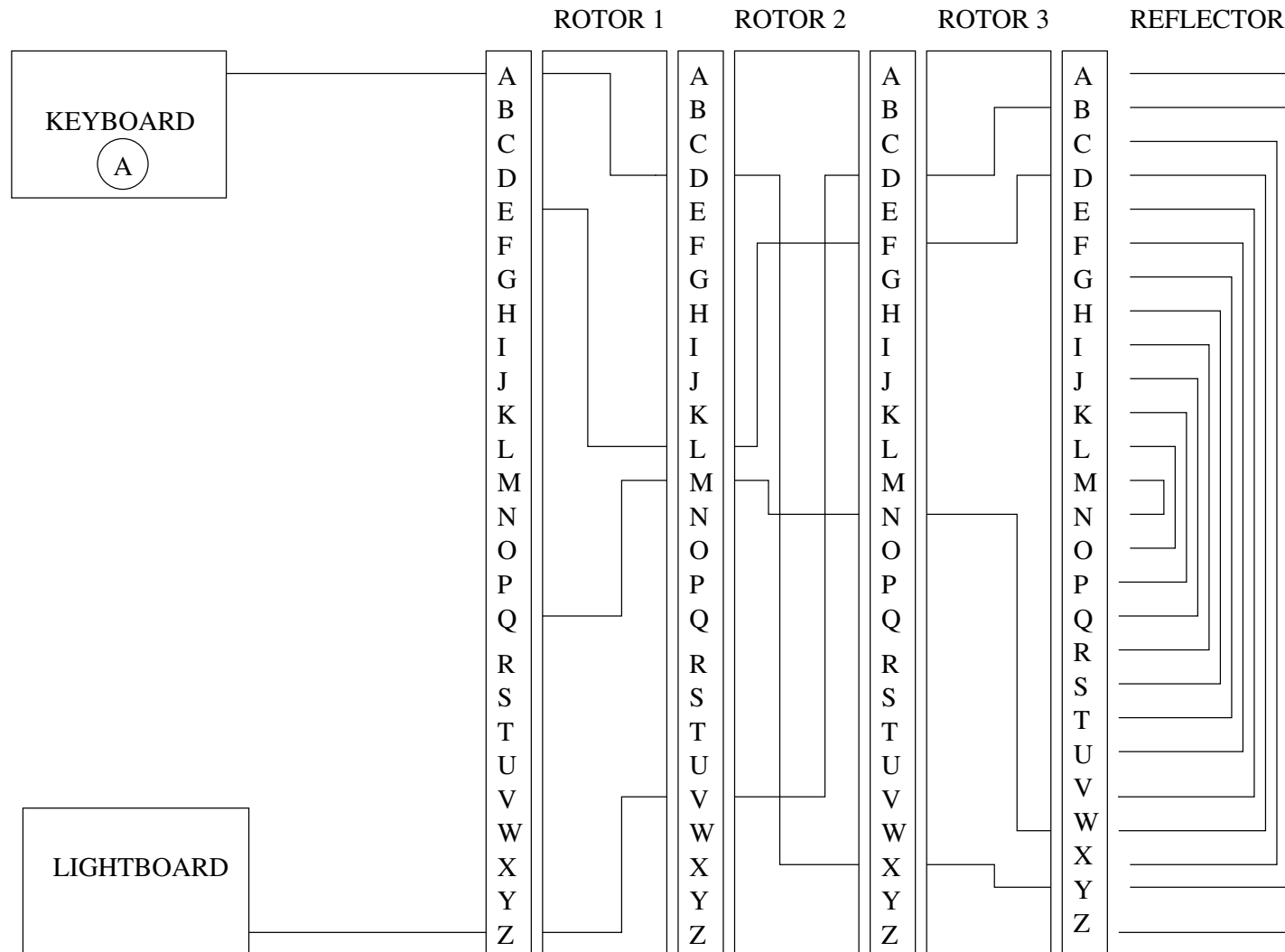
- Rotors start in any one of 26 positions.

Enigma Schematic



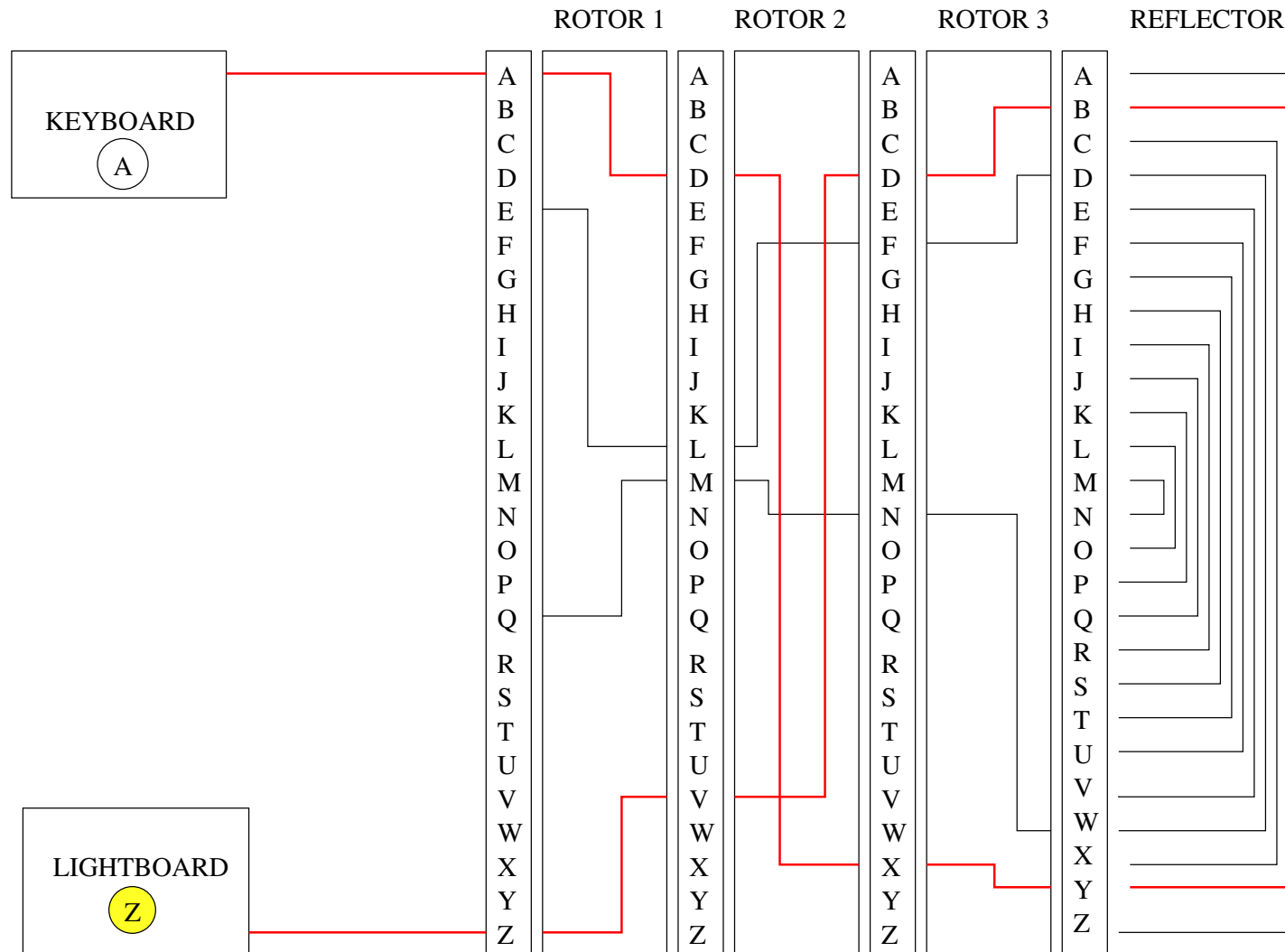
- Rotors start in any one of 26 positions.
- Rotors start in any one of $3! = 6$ orders.

Enigma Schematic



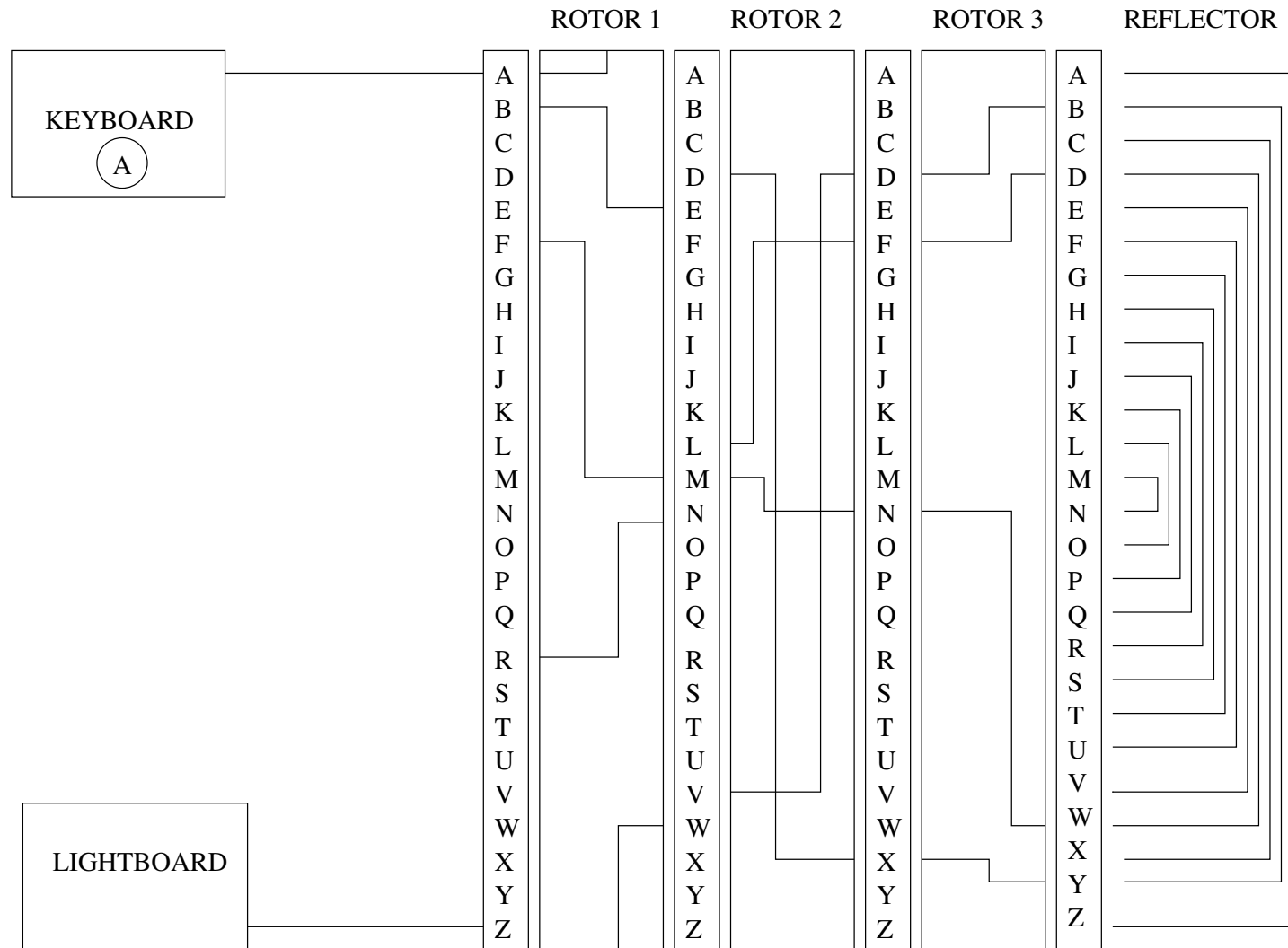
- Rotors start in any one of 26 positions.
- Rotors start in any one of $3! = 6$ orders.
- Reflector is fixed.

Enigma Schematic



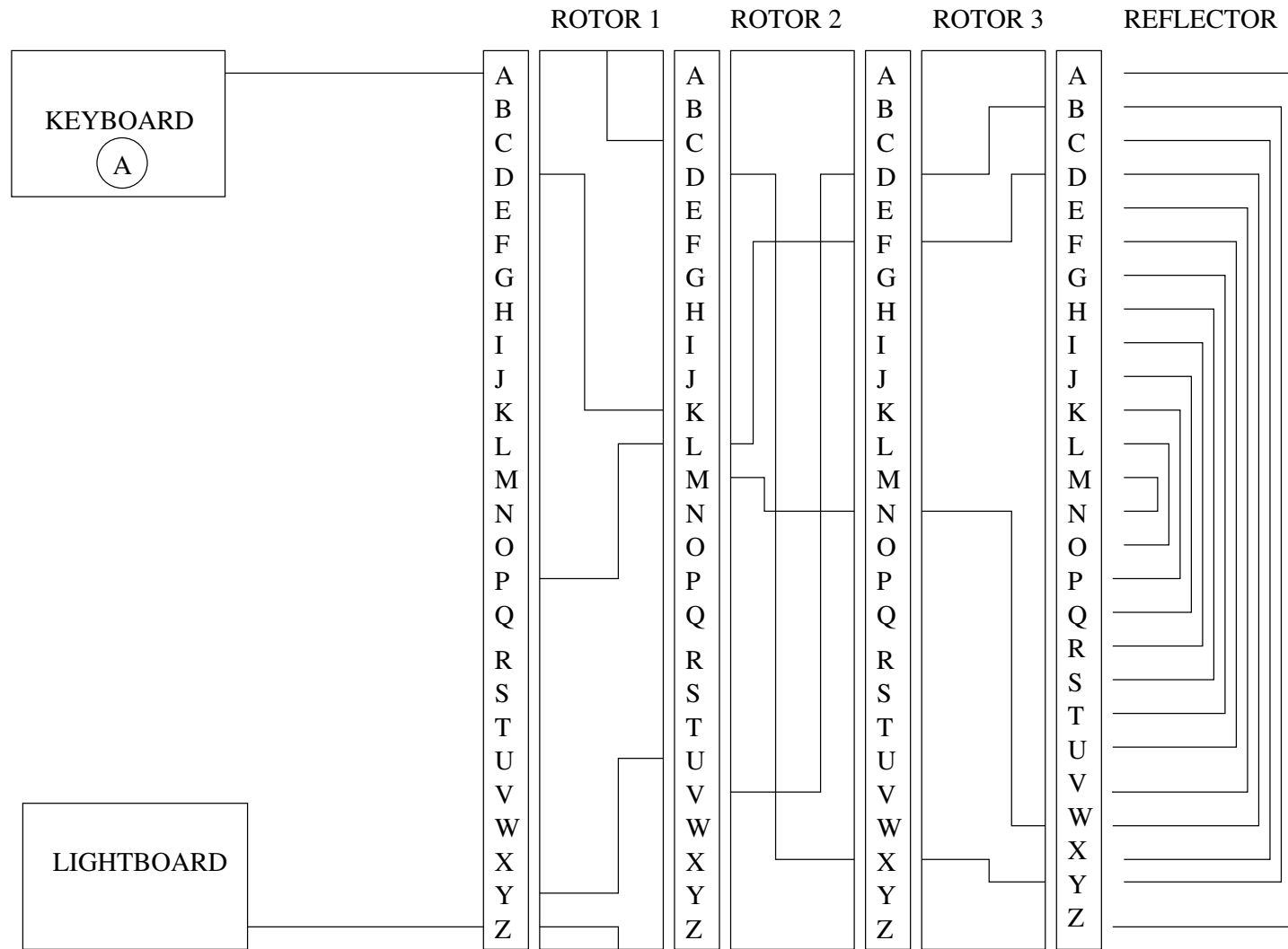
- A encrypts to Z.

Enigma Schematic



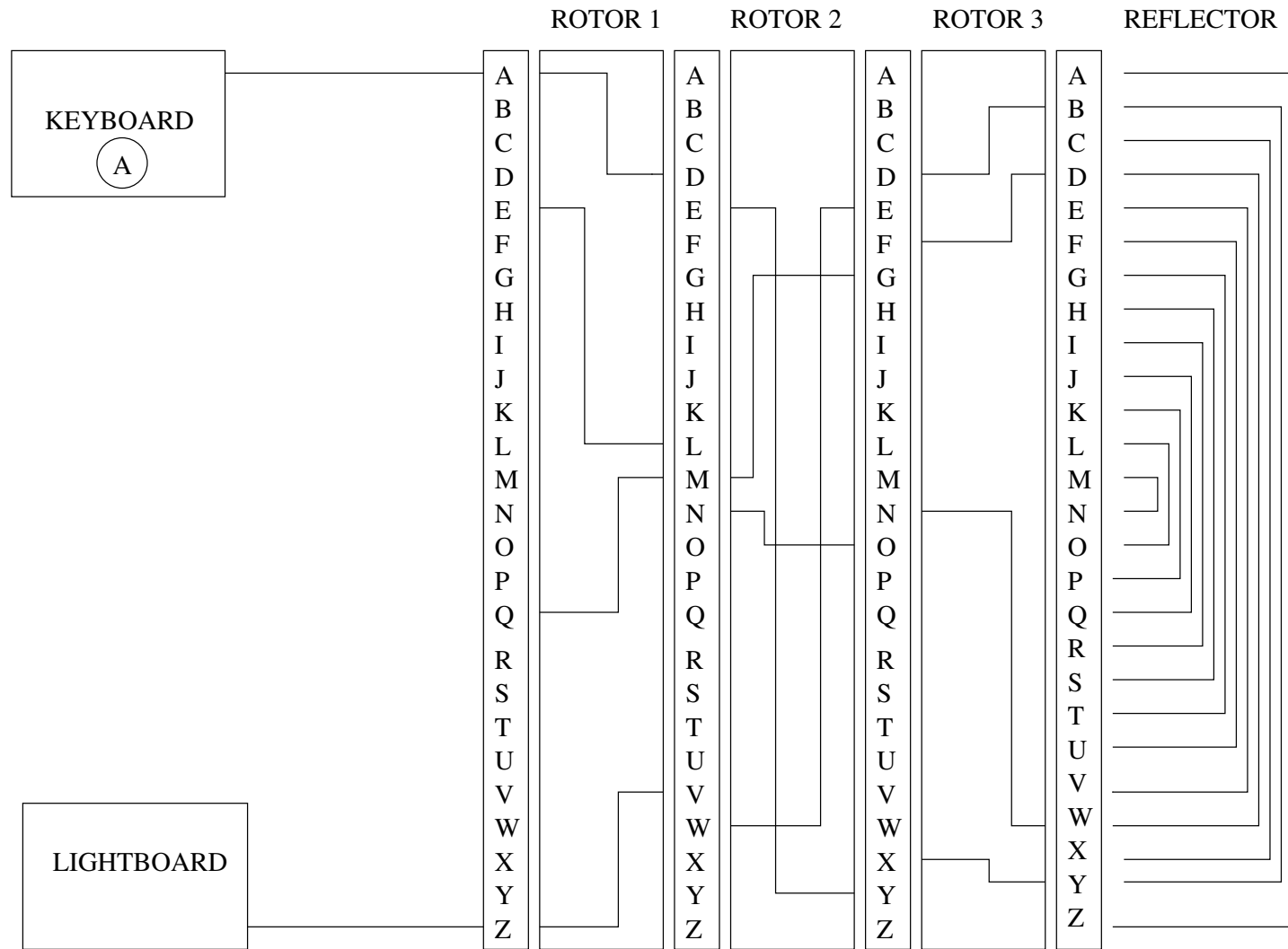
- A encrypts to Z.
- Rotor 1 moves after each letter is encrypted.

Enigma Schematic



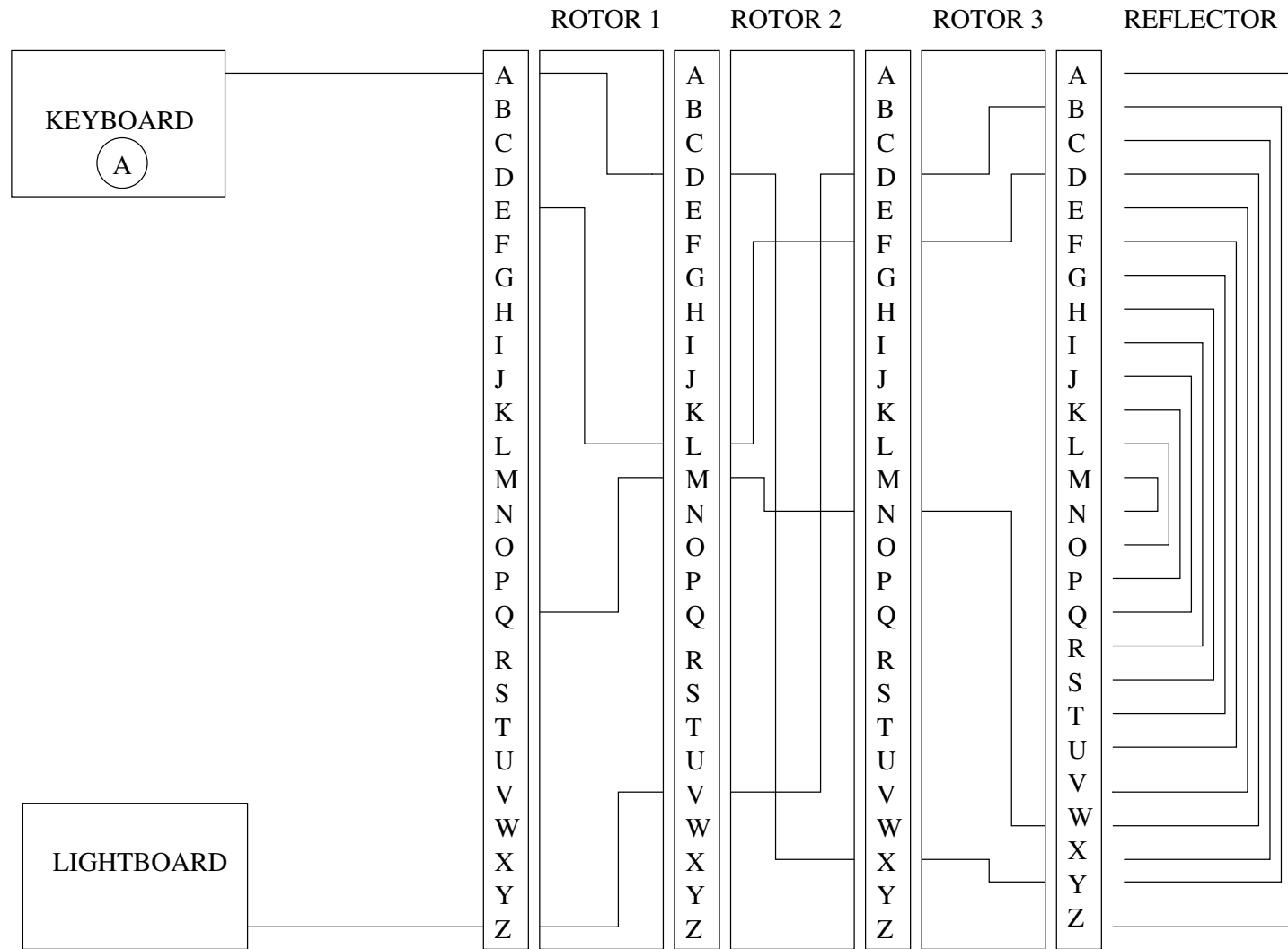
- A encrypts to Z.
- Rotor 1 moves after each letter is encrypted.
- After 25 encryptions ...

Enigma Schematic



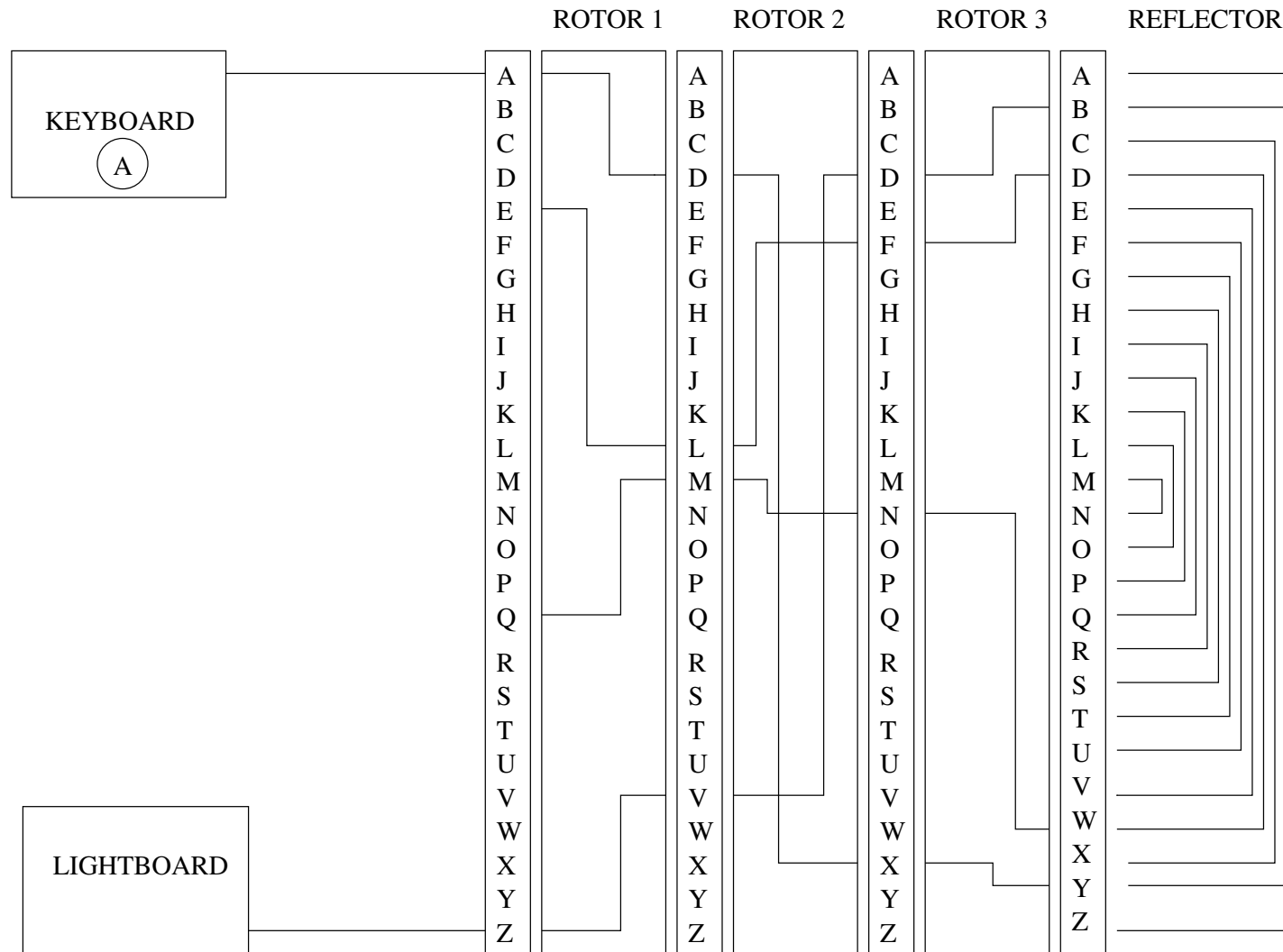
- Rotor 2 moves when Rotor 1 returns to starting position.

Enigma Schematic



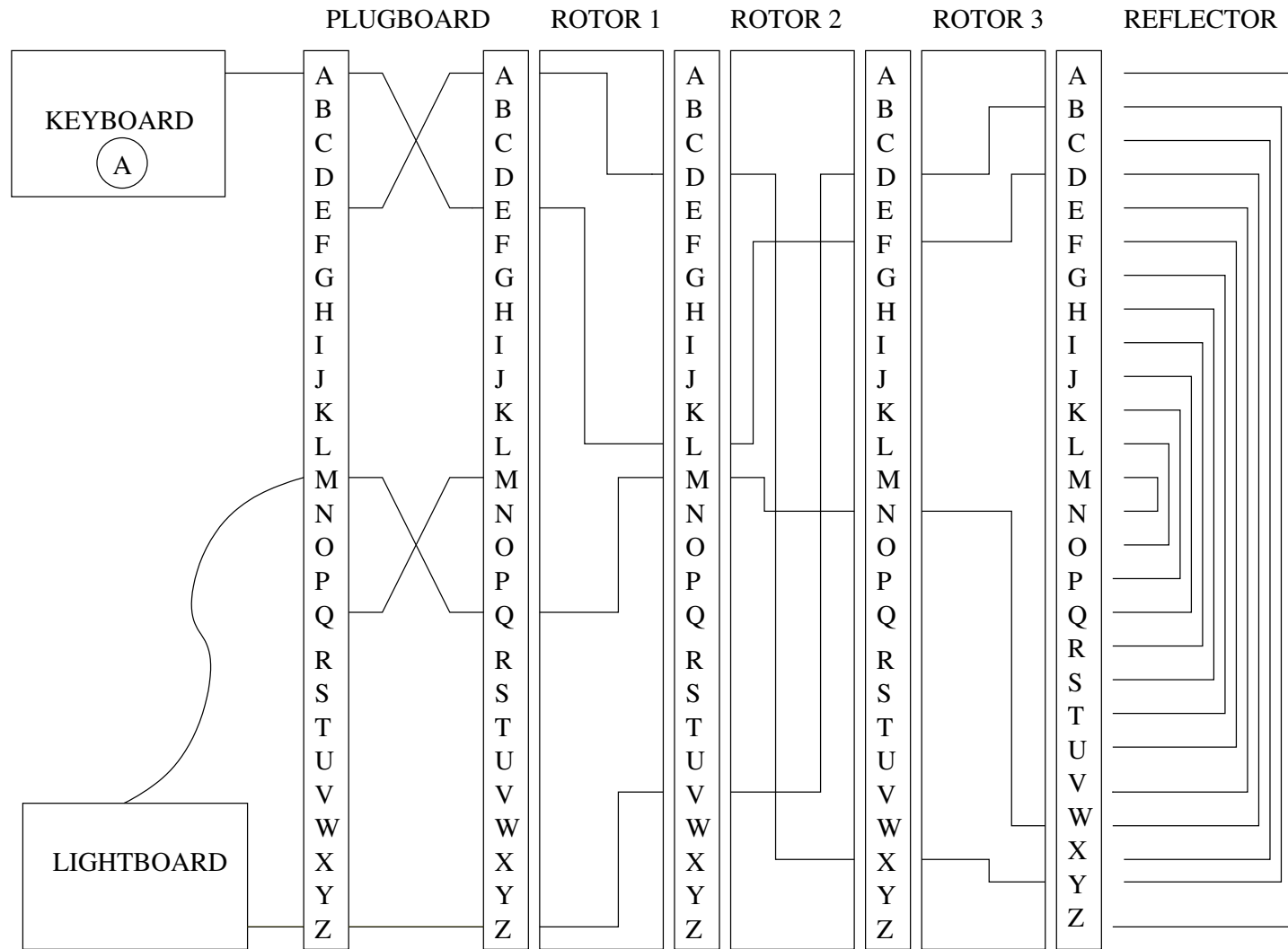
- Keysize is $26^3 \times 6 = 105,456$.

Enigma Schematic



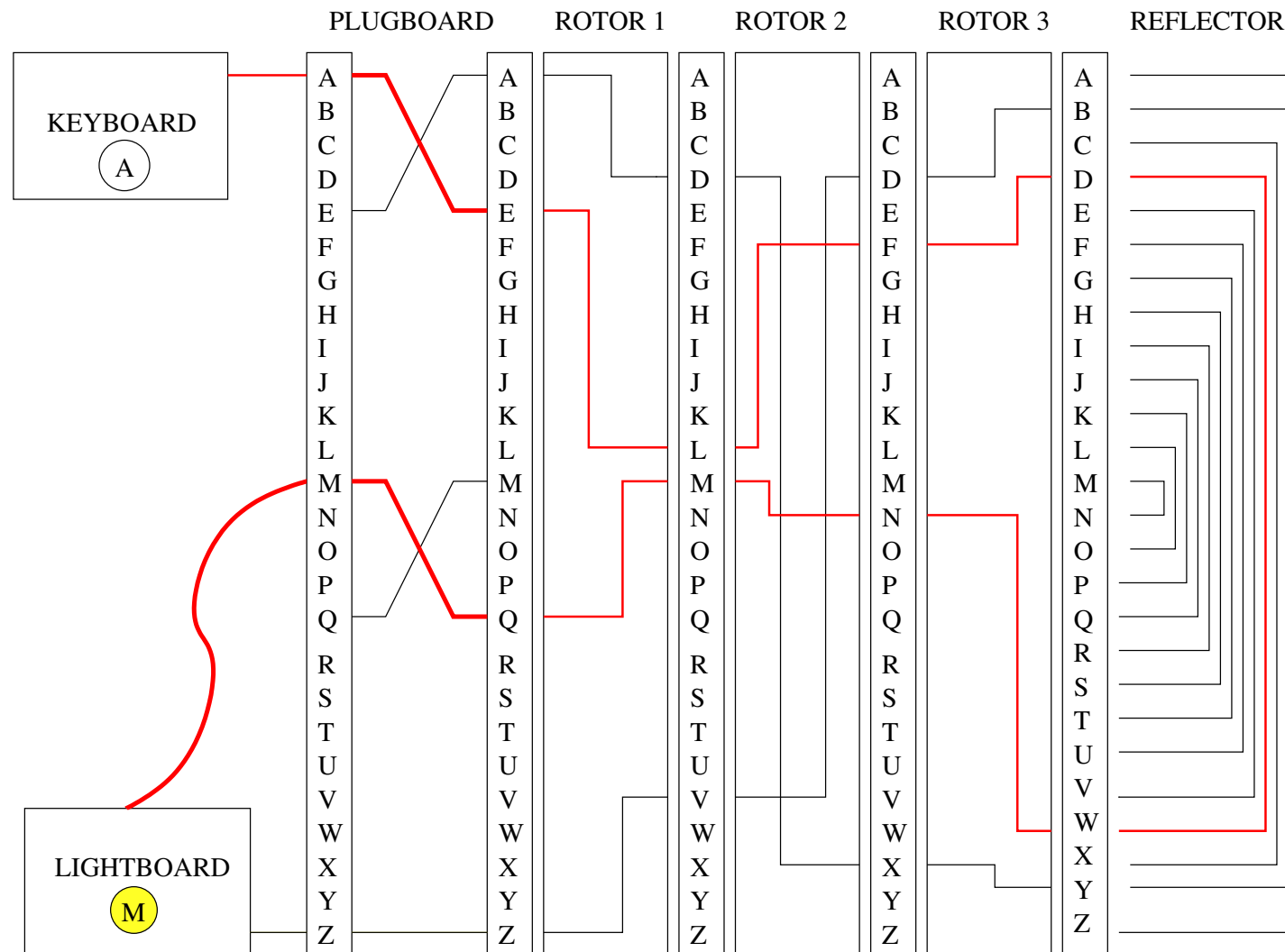
- Keysize is $26^3 \times 6 = 105,456$.
- Doesn't repeat until $26^3 = 17,576$ letters.

Enigma Schematic



- Plugboard: 6 pairs of letter swaps.

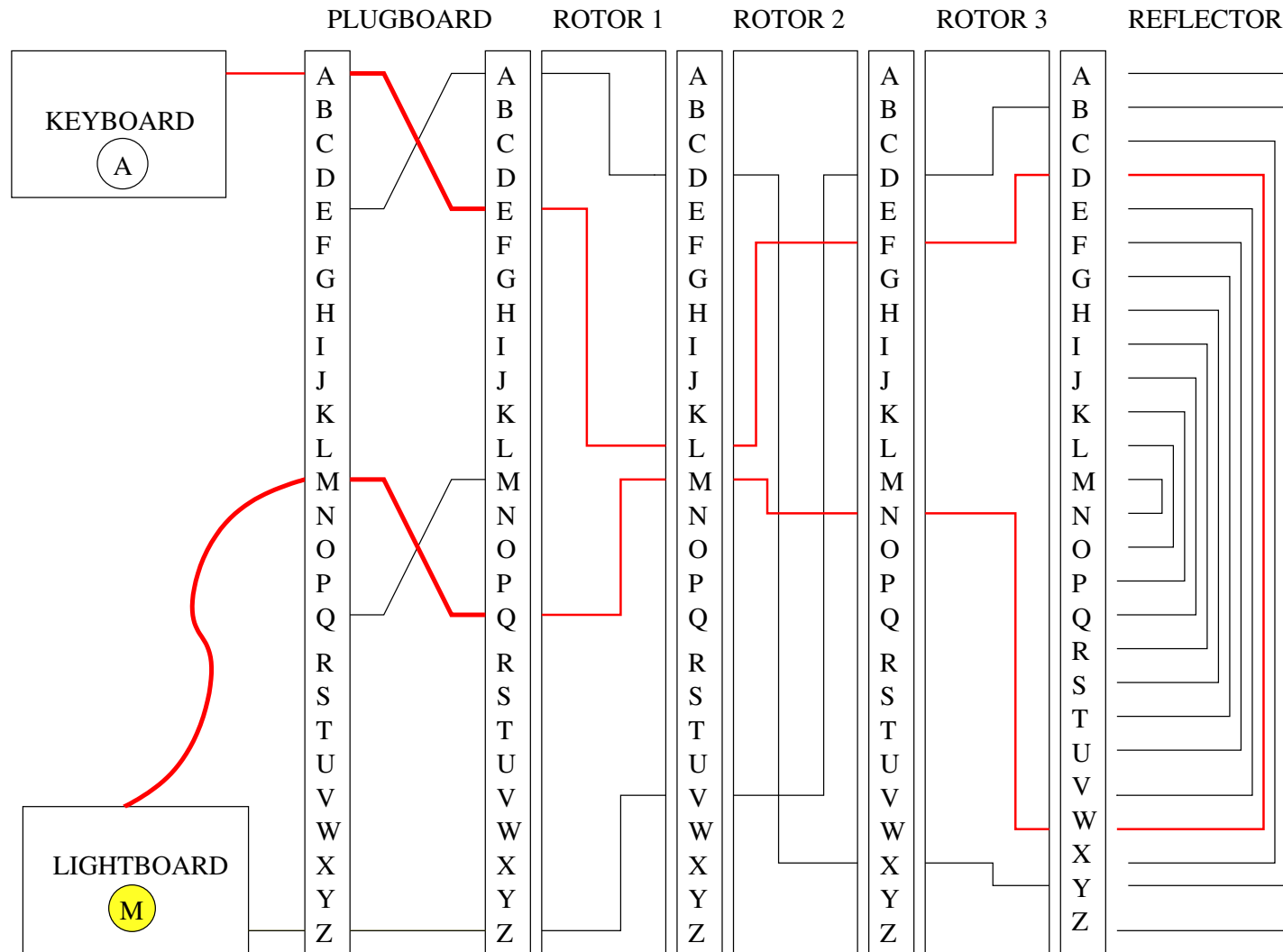
Enigma Schematic



- A encrypts to M.

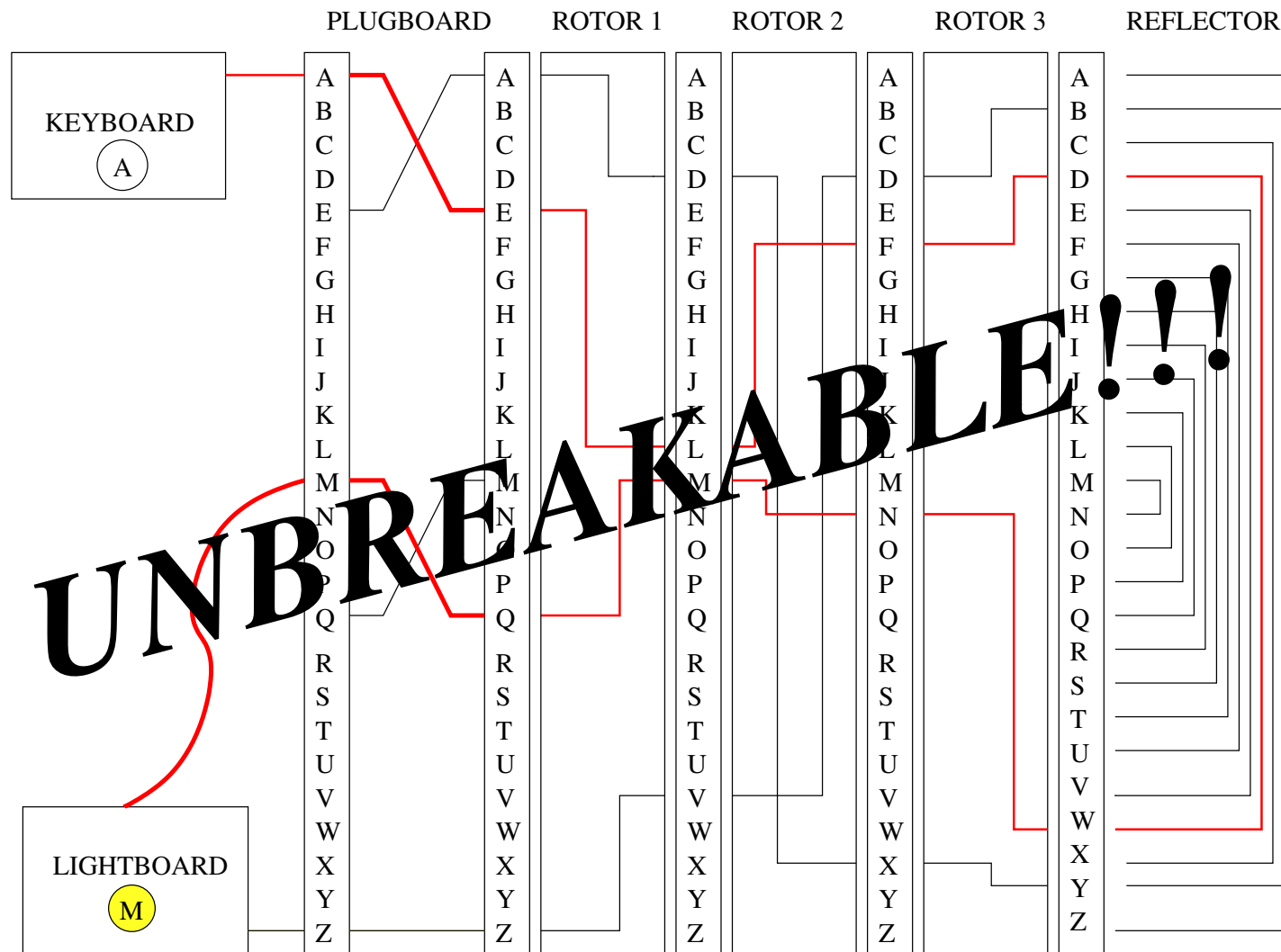
- Plugboard: 6 pairs of letter swaps.

Enigma Schematic



- A encrypts to M.
- Keysize is $26^3 \times 6 \times (100, 391, 791, 500) \approx 10^{16}$.
- Plugboard: 6 pairs of letter swaps.

Enigma Schematic



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- Due to post-war depression, widespread public use did not come to pass. Military sees no need for added expense.
- 1923: Britain publishes official history of WWI, boasting of great cryptographic triumphs.
- 1925: German military begins use of Enigma. Decade of unparalleled security begins...

The Enigma Protocol

- Daily Codebook contains **Day key**, consisting of:
 1. Rotor Order: 2-1-3
 2. Plugboard Swaps: A/V, B/R, S/U, N/W, D/P, C/Q.
 3. Rotor Settings: X-V-F

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 3. Rotor Settings: X-V-F
- A **Message key** is used per message to avoid depths during encryption:

ANFANF THISISTHEMESSAGE
Use 1,2,3 Use 1,2, and A-N-F

Espionage

- Hans-Thilo Schmidt

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Marian Rejewski

Cracking the Enigma

BOLJRVSQIGPQTMNWJRAKOBYTKMTTG
BBRQUPWLHSOLNFEQTHJOVXSWPAEWM
CWPBHKGABJOPHAXOYJIKXEGSBLZWB
QCOUMYYQGRKTNPSORSTOYHYASQGNV
IHFGFOTMINEDDXOYMKGGTXUQMJPKZ
CYDLCZZWGQAWZNHSKJSWPXNCQJZDP
VLROVJGLSDCPRLWHQTSSCHALESKFN
XIRZGYWUDJODMSPPSZBJEZJAEQA JG
PAGYOSILDHELQXKINYNYET

Cracking the Enigma

- The first six letters of every message on a given day are in depth!

Cracking the Enigma

- The first six letters of every message on a given day are in depth!
- In the same day, we have several other messages. Here are the first six characters of each of them:

BOLJRV	WKOTFI	JOSURM	EFKBOT	RBEDAP
TBHCA X	HWKSB T	YQDZNS	EBXBAB	KZXAQB
DABNUW	QFMQOF	WEOTSI	UWGMBN	WRBTJW
WLDTVS	ZYDKMS	FAREUC	XXHXKX	DGDNXS
NNSHDM	QKXQFB	CCZFLH	VCHVLX	ADPRWQ
XQUXNA	JHJUGY	TULCYV	PFYWOL	NQVHNG
YKIZFK	GGDGXS	BSXJEB	TITCTZ	SZALQR
KKDAFS	SSVLEG	IICITU	LPSYZM	OGKOXT
LXRYKC	MOXPRB	SLNLVE	KTFAID	XVAXHR
HFJSOY	JJQUCJ	DMWNPO	REJDSY	XUZXYH

Cracking the Enigma

Permutation	P_1	P_2	P_3	P_4	P_5	P_6
Plaintext	a	b	c	a	b	c
Ciphertext	B	O	L	J	R	V

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Permutation	P_1	P_2	P_3	P_4	P_5	P_6
Plaintext	a	b	c	a	b	c
Ciphertext	B	O	L	J	R	V

- Call the first permutation defined by this Enigma setting, P_1 , the second P_2 , and so forth, then we can determine that:
 1. P_1 : $a \iff B$
 2. P_2 : $b \iff O$
 3. P_3 : $c \iff L$
 4. P_4 : $a \iff J$
 5. P_5 : $b \iff R$
 6. P_6 : $c \iff V$

Cracking the Enigma

Permutation	P_1	P_2	P_3	P_4	P_5	P_6
Plaintext	a	b	c	a	b	c
Ciphertext	B	0	L	J	R	V

- Call the first permutation defined by this Enigma setting, P_1 , the second P_2 , and so forth, then we can determine that:
 1. P_1 : $a \iff B$
 2. P_2 : $b \iff 0$
 3. P_3 : $c \iff L$
 4. P_4 : $a \iff J$
 5. P_5 : $b \iff R$
 6. P_6 : $c \iff V$
- We don't know what a is, but we do know that P_1 links a to B, and P_4 links a to J. This leads to the Rejewski's breakthrough observation:

Cracking the Enigma

- The intercept BOLLJRV tells us that
 1. $P_4 \circ P_1(\text{B}) = \text{J}$,
 2. $P_5 \circ P_2(\text{O}) = \text{R}$,
 3. $P_6 \circ P_3(\text{L}) = \text{V}$.

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- The next intercept, WKOTFI, tells us that
 1. $P_4 \circ P_1(\text{W}) = \text{T}$,
 2. $P_5 \circ P_2(\text{K}) = \text{F}$,
 3. $P_6 \circ P_3(\text{O}) = \text{I}$.

Cracking the Enigma

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 1. $P_4 \circ P_1(\text{B})=\text{J}$,
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 3. $P_6 \circ P_3(\text{L})=\text{V}$.
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 2. $P_5 \circ P_2(\text{K})=\text{F}$,
 3. $P_6 \circ P_3(\text{O})=\text{I}$.
- We can use the entire set of intercepts to construct the following table of relationships:

Cracking the Enigma

The permutation $P_4 \circ P_1$

$P_4 \circ P_1$	A	B	C	D	E	F	G	H	I
	R	J	F	N	B	E	G	S	I
$P_4 \circ P_1$	J	K	L	M	N	O	P	Q	R
	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
	L	C	M	V	T	X	Z	K	

- BOLJRV WKOTFI JOSURM EFKBOT RBEDAP

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	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
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$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
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	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
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- BOLJRV WKOTFI JOSURM **E**FK**B**OT RBEDAP

Cracking the Enigma

The permutation $P_4 \circ P_1$

$P_4 \circ P_1$	A	B	C	D	E	F	G	H	I
	R	J	F	N	B	E	G	S	I
$P_4 \circ P_1$	J	K	L	M	N	O	P	Q	R
	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
	L	C	M	V	T	X	Z	K	

- BOLJRV WKOTFI JOSURM EFKBOT **R**BEDAP

Cracking the Enigma

The permutation $P_4 \circ P_1$

$P_4 \circ P_1$	A	B	C	D	E	F	G	H	I
	R	J	F	N	B	E	G	S	I
$P_4 \circ P_1$	J	K	L	M	N	O	P	Q	R
	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
	L	C	M	V	T	X	Z	K	

- BOLJRV WKOTFI JOSURM EFKBOT RBEDAP
- Rejewski observed cycles within this permutation.

Cracking the Enigma

The permutation $P_4 \circ P_1$

$P_4 \circ P_1$	A	B	C	D	E	F	G	H	I
	R	J	F	N	B	E	G	S	I
$P_4 \circ P_1$	J	K	L	M	N	O	P	Q	R
	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
	L	C	M	V	T	X	Z	K	

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A-->R

Cracking the Enigma

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$P_4 \circ P_1$	A	B	C	D	E	F	G	H	I
	R	J	F	N	B	E	G	S	I
$P_4 \circ P_1$	J	K	L	M	N	O	P	Q	R
	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
	L	C	M	V	T	X	Z	K	

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A-->R-->D

Cracking the Enigma

The permutation $P_4 \circ P_1$

$P_4 \circ P_1$	A	B	C	D	E	F	G	H	I
	R	J	F	N	B	E	G	S	I
$P_4 \circ P_1$	J	K	L	M	N	O	P	Q	R
	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
	L	C	M	V	T	X	Z	K	

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A-->R-->D-->N

Cracking the Enigma

The permutation $P_4 \circ P_1$

$P_4 \circ P_1$	A	B	C	D	E	F	G	H	I
	R	J	F	N	B	E	G	S	I
$P_4 \circ P_1$	J	K	L	M	N	O	P	Q	R
	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
	L	C	M	V	T	X	Z	K	

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A-->R-->D-->N-->H

Cracking the Enigma

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$P_4 \circ P_1$	A	B	C	D	E	F	G	H	I
	R	J	F	N	B	E	G	S	I
$P_4 \circ P_1$	J	K	L	M	N	O	P	Q	R
	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
	L	C	M	V	T	X	Z	K	

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A-->R-->D-->N-->H-->S

Cracking the Enigma

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$P_4 \circ P_1$	A	B	C	D	E	F	G	H	I
	R	J	F	N	B	E	G	S	I
$P_4 \circ P_1$	J	K	L	M	N	O	P	Q	R
	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
	L	C	M	V	T	X	Z	K	

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A-->R-->D-->N-->H-->S-->L

Cracking the Enigma

The permutation $P_4 \circ P_1$

$P_4 \circ P_1$	A	B	C	D	E	F	G	H	I
	R	J	F	N	B	E	G	S	I
$P_4 \circ P_1$	J	K	L	M	N	O	P	Q	R
	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
	L	C	M	V	T	X	Z	K	

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A-->R-->D-->N-->H-->S-->L-->Y

Cracking the Enigma

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$P_4 \circ P_1$	A	B	C	D	E	F	G	H	I
	R	J	F	N	B	E	G	S	I
$P_4 \circ P_1$	J	K	L	M	N	O	P	Q	R
	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
	L	C	M	V	T	X	Z	K	

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A-->R-->D-->N-->H-->S-->L-->Y-->Z

Cracking the Enigma

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$P_4 \circ P_1$	A	B	C	D	E	F	G	H	I
	R	J	F	N	B	E	G	S	I
$P_4 \circ P_1$	J	K	L	M	N	O	P	Q	R
	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
	L	C	M	V	T	X	Z	K	

- BOLJRV WKOTFI JOSURM EFKBOT RBEDAP
- Rejewski observed cycles within this permutation.

A-->R-->D-->N-->H-->S-->L-->Y-->Z-->K

Cracking the Enigma

The permutation $P_4 \circ P_1$

$P_4 \circ P_1$	A	B	C	D	E	F	G	H	I
	R	J	F	N	B	E	G	S	I
$P_4 \circ P_1$	J	K	L	M	N	O	P	Q	R
	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
	L	C	M	V	T	X	Z	K	

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	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
	L	C	M	V	T	X	Z	K	

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$P_4 \circ P_1$	A	B	C	D	E	F	G	H	I
	R	J	F	N	B	E	G	S	I
$P_4 \circ P_1$	J	K	L	M	N	O	P	Q	R
	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
	L	C	M	V	T	X	Z	K	

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	R	J	F	N	B	E	G	S	I
$P_4 \circ P_1$	J	K	L	M	N	O	P	Q	R
	U	A	Y	P	H	O	W	Q	D
$P_4 \circ P_1$	S	T	U	V	W	X	Y	Z	
	L	C	M	V	T	X	Z	K	

- BOLJRV WKOTFI JOSURM EFKBOT RBEDAP
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A → R → D → N → H → S → L → Y → Z → K → A

B → J → U → M → P → W → T → C → F → E → B

G → G I → I O → O Q → Q V → V X → X

- We say $P_4 \circ P_1$ has cycle structure 10-10-1-1-1-1-1-1.

Cracking the Enigma

The permutation $P_5 \circ P_2$

$P_5 \circ P_2$	A	B	C	D	E	F	G	H	I
	U	A	L	W	S	O	X	G	T
$P_5 \circ P_2$	J	K	L	M	N	O	P	Q	R
	C	F	V	P	D	R	Z	N	J
$P_5 \circ P_2$	S	T	U	V	W	X	Y	Z	
	E	I	Y	H	B	K	M	Q	

Cracking the Enigma

The permutation $P_5 \circ P_2$

$P_5 \circ P_2$	A	B	C	D	E	F	G	H	I
	U	A	L	W	S	O	X	G	T
$P_5 \circ P_2$	J	K	L	M	N	O	P	Q	R
	C	F	V	P	D	R	Z	N	J
$P_5 \circ P_2$	S	T	U	V	W	X	Y	Z	
	E	I	Y	H	B	K	M	Q	

- $P_5 \circ P_2$ has cycles:

A → U → Y → M → P → Z → Q → N → D → W → B → A

C → L → V → H → G → X → K → F → O → R → J → C

E → S → E

I → T → I

Cracking the Enigma

The permutation $P_5 \circ P_2$

$P_5 \circ P_2$	A	B	C	D	E	F	G	H	I
	U	A	L	W	S	O	X	G	T
$P_5 \circ P_2$	J	K	L	M	N	O	P	Q	R
	C	F	V	P	D	R	Z	N	J
$P_5 \circ P_2$	S	T	U	V	W	X	Y	Z	
	E	I	Y	H	B	K	M	Q	

- $P_5 \circ P_2$ has cycles:

A → U → Y → M → P → Z → Q → N → D → W → B → A

C → L → V → H → G → X → K → F → O → R → J → C

E → S → E

I → T → I

- $P_5 \circ P_2$ has cycle structure 11-11-2-2.

Cracking the Enigma

The permutation $P_6 \circ P_3$

$P_6 \circ P_3$	A	B	C	D	E	F	G	H	I
	R	W	U	S	P	D	N	X	K
$P_6 \circ P_3$	J	K	L	M	N	O	P	Q	R
	Y	T	V	F	E	I	Q	J	C
$P_6 \circ P_3$	S	T	U	V	W	X	Y	Z	
	M	Z	A	G	O	B	L	H	

Cracking the Enigma

The permutation $P_6 \circ P_3$

$P_6 \circ P_3$	A	B	C	D	E	F	G	H	I
	R	W	U	S	P	D	N	X	K
$P_6 \circ P_3$	J	K	L	M	N	O	P	Q	R
	Y	T	V	F	E	I	Q	J	C
$P_6 \circ P_3$	S	T	U	V	W	X	Y	Z	
	M	Z	A	G	O	B	L	H	

- $P_6 \circ P_3$ has cycles:

B → W → O → I → K → T → Z → H → X → B

E → P → Q → J → Y → L → V → G → N → E

A → R → C → U → A

D → S → M → F → D

Cracking the Enigma

The permutation $P_6 \circ P_3$

$P_6 \circ P_3$	A	B	C	D	E	F	G	H	I
	R	W	U	S	P	D	N	X	K
$P_6 \circ P_3$	J	K	L	M	N	O	P	Q	R
	Y	T	V	F	E	I	Q	J	C
$P_6 \circ P_3$	S	T	U	V	W	X	Y	Z	
	M	Z	A	G	O	B	L	H	

- $P_6 \circ P_3$ has cycles:

B → W → O → I → K → T → Z → H → X → B

E → P → Q → J → Y → L → V → G → N → E

A → R → C → U → A

D → S → M → F → D

- $P_6 \circ P_3$ has cycle structure 9-9-4-4.

Cracking the Enigma

- Rejewski recorded that this particular setting of Enigma (rotor orders, settings and plugboards) had the mathematical signature
(10-10-1-1-1-1-1-1, 11-11-2-2, 9-9-4-4).

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- Rejewski recorded that this particular setting of Enigma (rotor orders, settings and plugboards) had the mathematical signature
(10-10-1-1-1-1-1-1, 11-11-2-2, 9-9-4-4).
- This triple of cycle lengths is a fingerprint of the underlying Enigma setting, but which of the billions and billions of settings could it be?

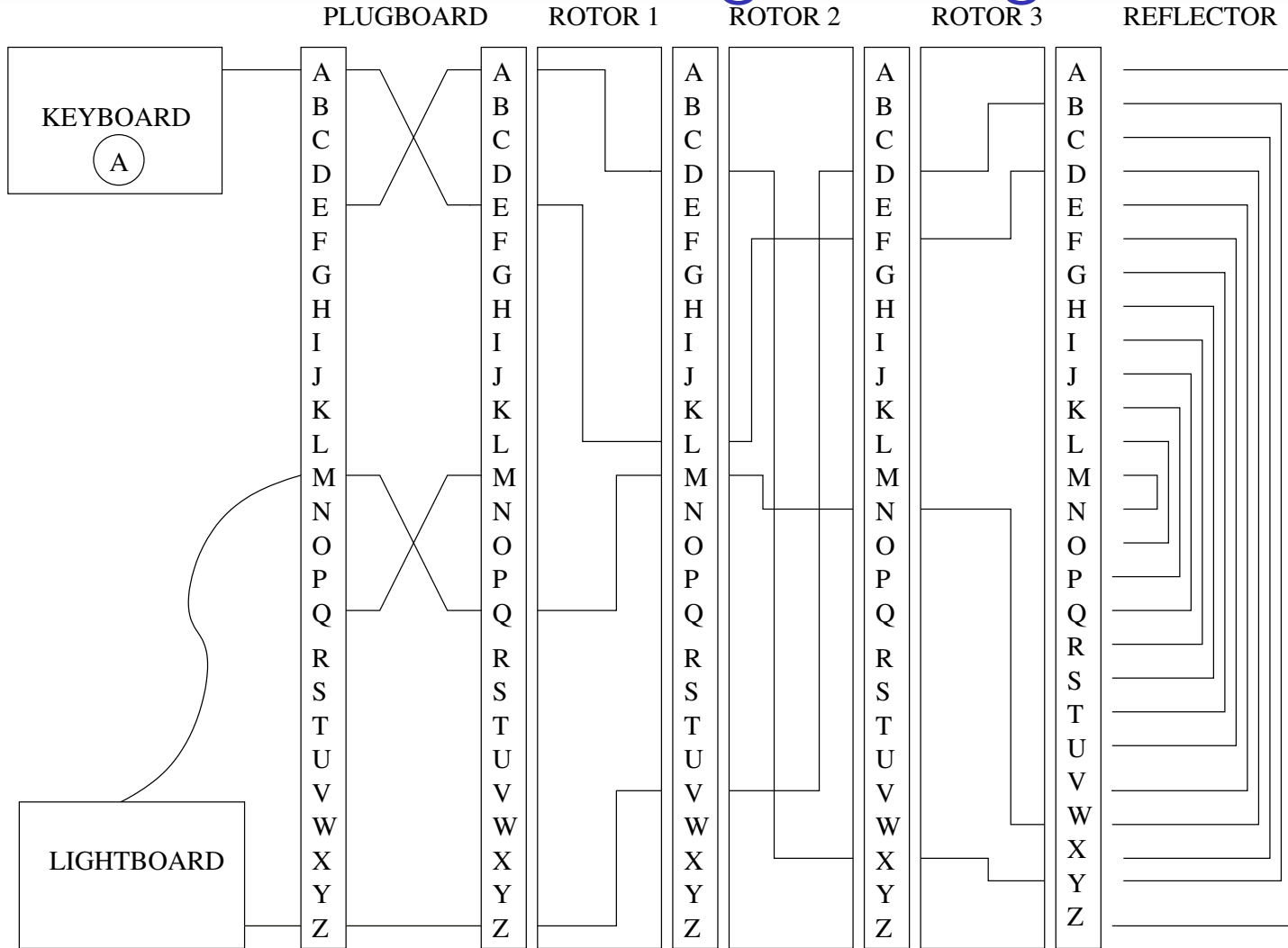
Cracking the Enigma

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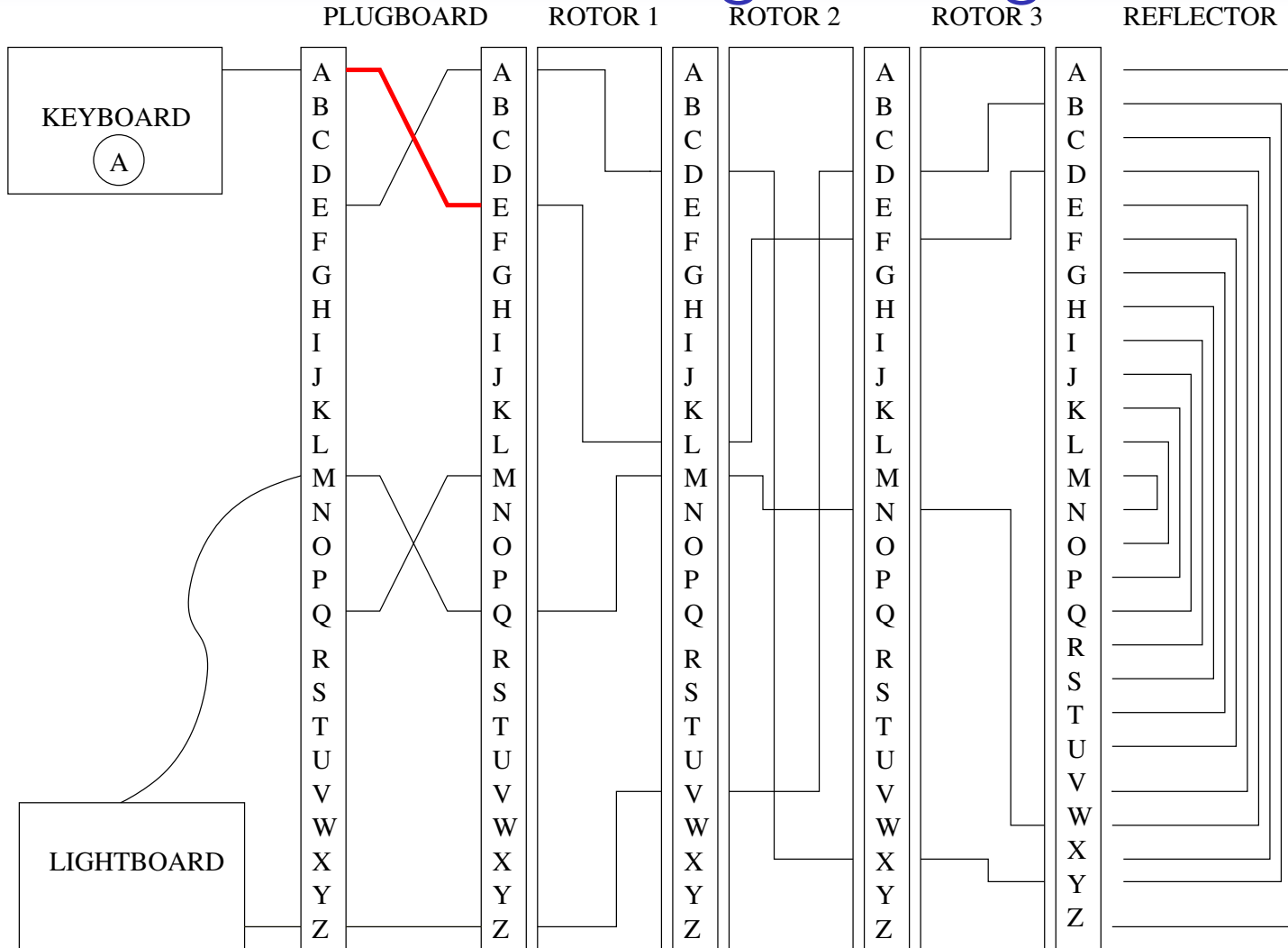
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- This triple of cycle lengths is a fingerprint of the underlying Enigma setting, but which of the billions and billions of settings could it be?
- **Recall:** $P_1 = P \circ \rho_1 \circ P$, where ρ_1 is the permutation defined by the initial rotor order and settings and P is the plugboard setting.

Cracking the Enigma

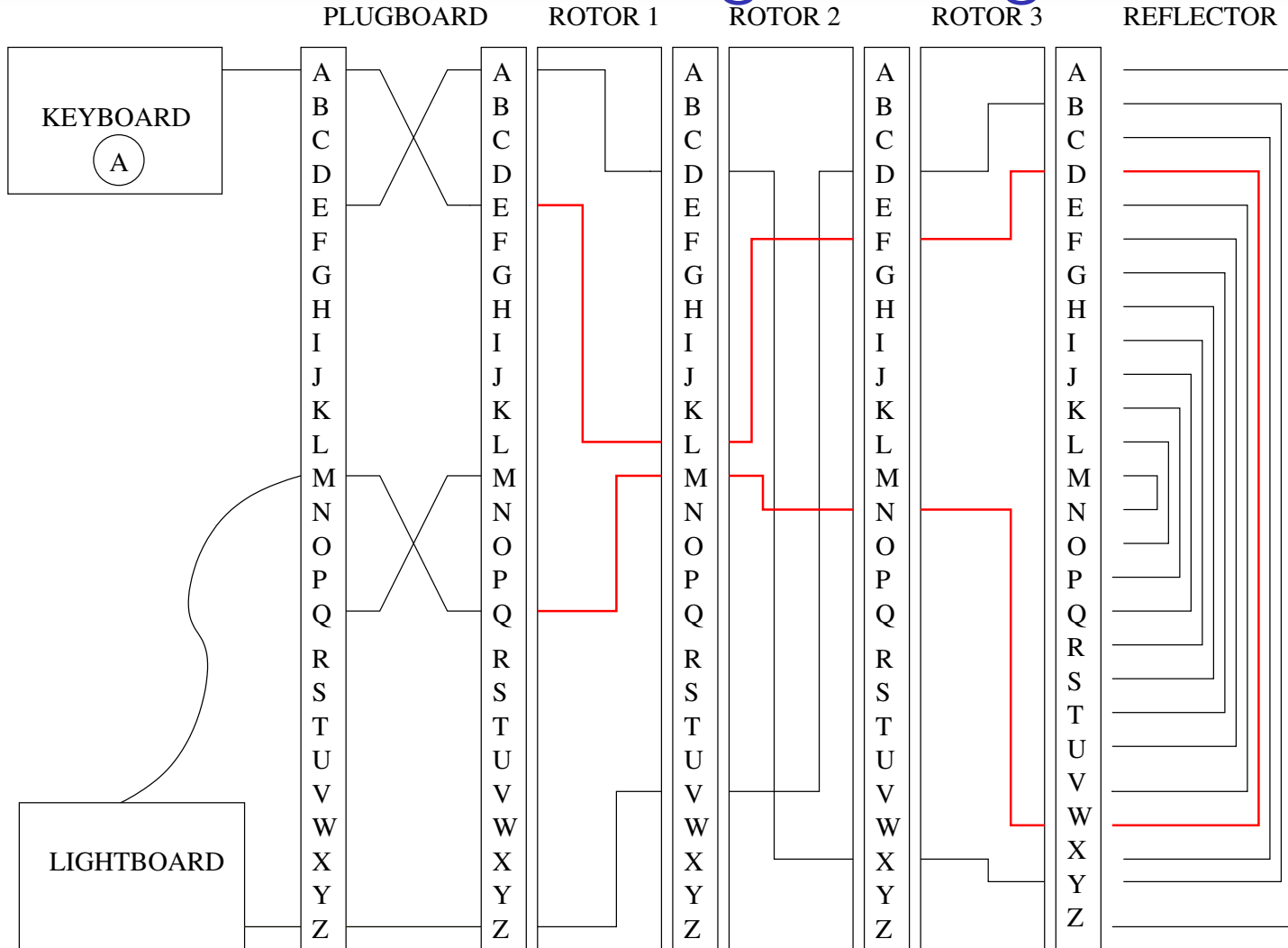


Cracking the Enigma



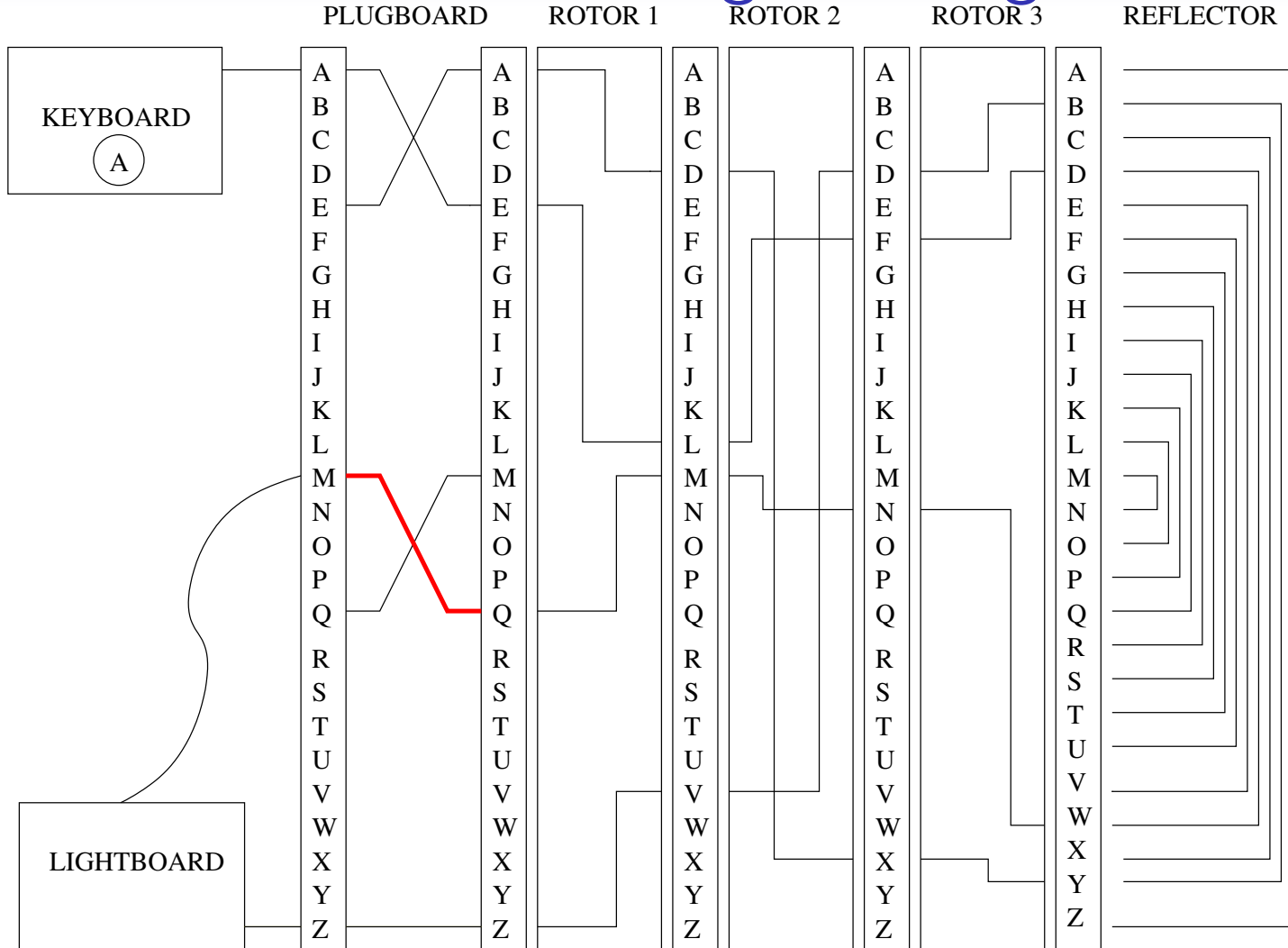
- First go through plugboard P .

Cracking the Enigma



- First go through plugboard P .
- Then pass through the rotor permutation ρ_1 .

Cracking the Enigma



- First go through plugboard P .
- Then pass through the rotor permutation ρ_1 .
- Pass again through $P = P^{-1}$.

Cracking the Enigma

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(10-10-1-1-1-1-1-1, 11-11-2-2, 9-9-4-4).
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- This triple of cycle lengths is a fingerprint of the underlying Enigma setting, but which of the billions and billions of settings could it be?
- **Recall:** $P_1 = P \circ \rho_1 \circ P$, where ρ_1 is the permutation defined by the initial rotor order and settings and P is the plugboard setting.
- Likewise, $P_4 = P \circ \rho_4 \circ P$.

Cracking the Enigma

Key Observation

$$\begin{aligned}P_4 \circ P_1 &= (P \circ \rho_4 \circ P) \circ (P \circ \rho_1 \circ P) \\&= P \circ \rho_4 \circ (P \circ P) \circ \rho_1 \circ P \\&= P \circ \rho_4 \circ \rho_1 \circ P \\&= P \circ (\rho_4 \circ \rho_1) \circ P.\end{aligned}$$

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- $P_4 \circ P_1$ is the result of conjugating $\rho_4 \circ \rho_1$ by the plugboard P .

Cracking the Enigma

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- $P_4 \circ P_1$ is the result of conjugating $\rho_4 \circ \rho_1$ by the plugboard P .
- The cycle structure of $P_4 \circ P_1$ is identical to that of $\rho_4 \circ \rho_1$.

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- $P_4 \circ P_1$ is the result of conjugating $\rho_4 \circ \rho_1$ by the plugboard P .
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- But $\rho_4 \circ \rho_1$ **involves only the rotors and reflector!**

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- $P_4 \circ P_1$ is the result of conjugating $\rho_4 \circ \rho_1$ by the plugboard P .
- The cycle structure of $P_4 \circ P_1$ is identical to that of $\rho_4 \circ \rho_1$.
- But $\rho_4 \circ \rho_1$ **involves only the rotors and reflector!**
- The cycle structure of $P_4 \circ P_1$ is **independent of the plugboard!**

Cracking the Enigma

- Recall there are only $6 \times 26^3 = 105,456$ settings determined by the rotor orders and starting positions.

Cracking the Enigma

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- Rejewski and his colleagues spent an entire year cataloguing the signature for each of the 105,456 starting positions.

Cracking the Enigma

- Recall there are only $6 \times 26^3 = 105,456$ settings determined by the rotor orders and starting positions.
- Rejewski and his colleagues spent an entire year cataloguing the signature for each of the 105,456 starting positions.
- The beginning of this catalogue might have looked like this:

Cracking the Enigma

Rotor order: 1 2 3: Setting: AAA
13-13- 12-12-1-1- 12-12-1-1-

Rotor order: 1 2 3: Setting: BAA
12-12-1-1- 12-12-1-1- 11-11-2-2-

Rotor order: 1 2 3: Setting: CAA
12-12-1-1- 11-11-2-2- 12-12-1-1-

Rotor order: 1 2 3: Setting: DAA
11-11-2-2- 12-12-1-1- 13-13-

Rotor order: 1 2 3: Setting: EAA
12-12-1-1- 13-13- 13-13-

Rotor order: 1 2 3: Setting: FAA
13-13- 13-13- 4-4-3-3-3-3-2-2-1-1-

Rotor order: 1 2 3: Setting: GAA
13-13- 4-4-3-3-3-3-2-2-1-1- 6-6-5-5-2-2-

Rotor order: 1 2 3: Setting: HAA
4-4-3-3-3-3-2-2-1-1- 6-6-5-5-2-2- 13-13-

The Final Assault

- Find (10-10-1-1-1-1-1-1, 11-11-2-2, 9-9-4-4) in the library.

The Final Assault

- Find (10-10-1-1-1-1-1-1, 11-11-2-2, 9-9-4-4) in the library.
- We find the following entry:
Rotor order: 2 3 1: Setting: ZQP.

The Final Assault

- Find (10-10-1-1-1-1-1-1, 11-11-2-2, 9-9-4-4) in the library.
- We find the following entry:
Rotor order: 2 3 1: Setting: ZQP.
- With model Enigmas, completely construct the first six permutations, ρ_1, \dots, ρ_6 , defined by these settings, **with no plugboard.**

The Final Assault

Rotor order: 2 3 1 Setting: Z Q P						
Perm.	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
A	M	V	Q	G	L	J
B	C	O	N	I	C	F
C	B	W	M	J	B	E
D	V	F	H	V	S	V
E	N	Z	Z	L	U	C
F	I	D	X	N	T	B
G	Y	L	L	A	P	X
H	W	Q	D	T	N	K
I	F	J	U	B	R	Z
J	Z	I	T	C	W	A
K	U	U	Y	M	Y	H
L	R	G	G	E	A	Q
M	A	P	C	K	X	R

Rotor order: 2 3 1 Setting: Z Q P						
Perm.	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
N	E	R	B	F	H	W
O	X	B	V	X	Z	P
P	Q	M	W	Q	G	O
Q	P	H	A	P	V	L
R	L	N	S	S	I	M
S	T	T	R	R	D	U
T	S	S	J	H	F	Y
U	K	K	I	Z	E	S
V	D	A	O	D	Q	D
W	H	C	P	Y	J	N
X	O	Y	F	O	M	G
Y	G	X	K	W	K	T
Z	J	E	E	U	O	I

The Final Assault

Rotor order: 2 3 1 Setting: Z Q P						
Perm.	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
A	M	V	Q	G	L	J
B	C	O	N	I	C	F
C	B	W	M	J	B	E
D	V	F	H	V	S	V
E	N	Z	Z	L	U	C
F	I	D	X	N	T	B
G	Y	L	L	A	P	X
H	W	Q	D	T	N	K
I	F	J	U	B	R	Z
J	Z	I	T	C	W	A
K	U	U	Y	M	Y	H
L	R	G	G	E	A	Q
M	A	P	C	K	X	R

Rotor order: 2 3 1 Setting: Z Q P						
Perm.	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
N	E	R	B	F	H	W
O	X	B	V	X	Z	P
P	Q	M	W	Q	G	O
Q	P	H	A	P	V	L
R	L	N	S	S	I	M
S	T	T	R	R	D	U
T	S	S	J	H	F	Y
U	K	K	I	Z	E	S
V	D	A	O	D	Q	D
W	H	C	P	Y	J	N
X	O	Y	F	O	M	G
Y	G	X	K	W	K	T
Z	J	E	E	U	O	I

- It is elementary to produce this table.

The Final Assault

Rotor order: 2 3 1 Setting: Z Q P						
Perm.	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
A	M	V	Q	G	L	J
B	C	O	N	I	C	F
C	B	W	M	J	B	E
D	V	F	H	V	S	V
E	N	Z	Z	L	U	C
F	I	D	X	N	T	B
G	Y	L	L	A	P	X
H	W	Q	D	T	N	K
I	F	J	U	B	R	Z
J	Z	I	T	C	W	A
K	U	U	Y	M	Y	H
L	R	G	G	E	A	Q
M	A	P	C	K	X	R

Rotor order: 2 3 1 Setting: Z Q P						
Perm.	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
N	E	R	B	F	H	W
O	X	B	V	X	Z	P
P	Q	M	W	Q	G	O
Q	P	H	A	P	V	L
R	L	N	S	S	I	M
S	T	T	R	R	D	U
T	S	S	J	H	F	Y
U	K	K	I	Z	E	S
V	D	A	O	D	Q	D
W	H	C	P	Y	J	N
X	O	Y	F	O	M	G
Y	G	X	K	W	K	T
Z	J	E	E	U	O	I

- It is elementary to produce this table.
- Set rotors in specified position. Hit A six times to produce first row. Get two entries for each encryption!

The Final Assault

Rotor order: 2 3 1 Setting: Z Q P						
Perm.	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
A	M	V	Q	G	L	J
B	C	O	N	I	C	F
C	B	W	M	J	B	E
D	V	F	H	V	S	V
E	N	Z	Z	L	U	C
F	I	D	X	N	T	B
G	Y	L	L	A	P	X
H	W	Q	D	T	N	K
I	F	J	U	B	R	Z
J	Z	I	T	C	W	A
K	U	U	Y	M	Y	H
L	R	G	G	E	A	Q
M	A	P	C	K	X	R

Rotor order: 2 3 1 Setting: Z Q P						
Perm.	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
N	E	R	B	F	H	W
O	X	B	V	X	Z	P
P	Q	M	W	Q	G	O
Q	P	H	A	P	V	L
R	L	N	S	S	I	M
S	T	T	R	R	D	U
T	S	S	J	H	F	Y
U	K	K	I	Z	E	S
V	D	A	O	D	Q	D
W	H	C	P	Y	J	N
X	O	Y	F	O	M	G
Y	G	X	K	W	K	T
Z	J	E	E	U	O	I

- It is elementary to produce this table.
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The Final Assault

Rotor order: 2 3 1 Setting: Z Q P						
Perm.	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
A	M	V	Q	G	L	J
B	C	O	N	I	C	F
C	B	W	M	J	B	E
D	V	F	H	V	S	V
E	N	Z	Z	L	U	C
F	I	D	X	N	T	B
G	Y	L	L	A	P	X
H	W	Q	D	T	N	K
I	F	J	U	B	R	Z
J	Z	I	T	C	W	A
K	U	U	Y	M	Y	H
L	R	G	G	E	A	Q
M	A	P	C	K	X	R

Rotor order: 2 3 1 Setting: Z Q P						
Perm.	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
N	E	R	B	F	H	W
O	X	B	V	X	Z	P
P	Q	M	W	Q	G	O
Q	P	H	A	P	V	L
R	L	N	S	S	I	M
S	T	T	R	R	D	U
T	S	S	J	H	F	Y
U	K	K	I	Z	E	S
V	D	A	O	D	Q	D
W	H	C	P	Y	J	N
X	O	Y	F	O	M	G
Y	G	X	K	W	K	T
Z	J	E	E	U	O	I

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Rotor order: 2 3 1 Setting: Z Q P						
Perm.	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
A	M	V	Q	G	L	J
B	C	O	N	I	C	F
C	B	W	M	J	B	E
D	V	F	H	V	S	V
E	N	Z	Z	L	U	C
F	I	D	X	N	T	B
G	Y	L	L	A	P	X
H	W	Q	D	T	N	K
I	F	J	U	B	R	Z
J	Z	I	T	C	W	A
K	U	U	Y	M	Y	H
L	R	G	G	E	A	Q
M	A	P	C	K	X	R

Rotor order: 2 3 1 Setting: Z Q P						
Perm.	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
N	E	R	B	F	H	W
O	X	B	V	X	Z	P
P	Q	M	W	Q	G	O
Q	P	H	A	P	V	L
R	L	N	S	S	I	M
S	T	T	R	R	D	U
T	S	S	J	H	F	Y
U	K	K	I	Z	E	S
V	D	A	O	D	Q	D
W	H	C	P	Y	J	N
X	O	Y	F	O	M	G
Y	G	X	K	W	K	T
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The Final Assault

Rotor order: 2 3 1 Setting: Z Q P						
Perm.	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
A	M	V	Q	G	L	J
B	C	O	N	I	C	F
C	B	W	M	J	B	E
D	V	F	H	V	S	V
E	N	Z	Z	L	U	C
F	I	D	X	N	T	B
G	Y	L	L	A	P	X
H	W	Q	D	T	N	K
I	F	J	U	B	R	Z
J	Z	I	T	C	W	A
K	U	U	Y	M	Y	H
L	R	G	G	E	A	Q
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T	S	S	J	H	F	Y
U	K	K	I	Z	E	S
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G	Y	L	L	A	P	X
H	W	Q	D	T	N	K
I	F	J	U	B	R	Z
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G	Y	L	L	A	P	X
H	W	Q	D	T	N	K
I	F	J	U	B	R	Z
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Recovering the Plugboard

- From this table, construct the cycles for the three permutations $\rho_4 \circ \rho_1$, $\rho_5 \circ \rho_2$, and $\rho_6 \circ \rho_3$:

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F	I	D	X	N	T	B
G	Y	L	L	A	P	X
H	W	Q	D	T	N	K
I	F	J	U	B	R	Z
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- For instance $\rho_1(M) = A$ and $\rho_4(A) = G$, so $\rho_4 \circ \rho_1(M) = G$.

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$$P_4 \circ P_1((P(\alpha))) = P \circ \rho_4 \circ \rho_1 \circ P((P(\alpha)))$$

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Recovering the Plugboard

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Recovering the Plugboard

Corollary

If $\rho_4 \circ \rho_1$ has a cycle $(\alpha_1 \alpha_2 \dots \alpha_n)$, then $P_4 \circ P_1$ has a cycle $(P(\alpha_1) P(\alpha_2) \dots P(\alpha_n))$.

Recovering the Plugboard

Corollary

If $\rho_4 \circ \rho_1$ has a cycle $(\alpha_1 \alpha_2 \dots \alpha_n)$, then $P_4 \circ P_1$ has a cycle $(P(\alpha_1) P(\alpha_2) \dots P(\alpha_n))$.

- This establishes the fact that $\rho_4 \circ \rho_1$ and $P_4 \circ P_1$ have identical cycle structures.

Recovering the Plugboard

1. $\rho_4 \circ \rho_1 =$

(MGWTREFBJU) (AKZCINLSHY) (P) (D) (O) (Q) (V) (X)

Recovering the Plugboard

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- Because the plugboard has 14 fixed points, we can look for common letter groups within cycles.

Recovering the Plugboard

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- The adjacency of letters in corresponding cycles suggests the proper alignment.

Recovering the Plugboard

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(ARDNHSLYZK) (**BJU**PWTCFE) (I) (O) (Q) (V) (X) (G)

- Because the plugboard has 14 fixed points, we can look for common letter groups within cycles.
- The adjacency of letters in corresponding cycles suggests the proper alignment.
- For instance, consider the letters **BJU** in $\rho_4 \circ \rho_1$ and $P_4 \circ P_1$.

Recovering the Plugboard

$\rho_4 \circ \rho_1 \rightarrow$ (BJUMGWTRF) (CINLSHYAKZ) (P) (D) (O) (Q) (V) (X)
 $P_4 \circ P_1 \rightarrow$ (BJUMPWTCFE) (RDNHSLYZKA) (G) (I) (O) (Q) (V) (X)

Recovering the Plugboard

$$\begin{array}{rcc} & \downarrow & \downarrow\downarrow \\ \rho_4 \circ \rho_1 & \rightarrow & (\text{BJUMGWTREF}) (\text{CINLSHYAKZ}) (\text{P}) (\text{D}) (\text{O}) (\text{Q}) (\text{V}) (\text{X}) \\ P_4 \circ P_1 & \rightarrow & (\text{BJUMPWTCFE}) (\text{RDNHSLYZKA}) (\text{G}) (\text{I}) (\text{O}) (\text{Q}) (\text{V}) (\text{X}) \\ & \uparrow & \uparrow\uparrow \end{array}$$

- The leftmost 11-cycles immediately yield the swaps R/C, G/P, and E/F.

Recovering the Plugboard

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Recovering the Plugboard

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- This immediately yields I/D, L/H, and A/Z. This completes the full recovery of all six letter swaps.

Recovering the Plugboard

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- This immediately yields I/D, L/H, and A/Z. This completes the full recovery of all six letter swaps.
- Having determined the rotor order:
2-3-1, the settings: Z-Q-P, and the plugboard swaps: G/P, I/D, A/Z, E/F, C/R, H/L, the message can be fully decrypted.