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4. McEliece public key cryptosystem is a candidate for NIST's quantum-resistant public key challenge.

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1. Modern Crypto is able to draw upon math already known.
2. Many protocols use elementary math since complicated math might be harder to code up and may have larger constants.

# A Long Aside: Error Correcting Codes

# Intentional Error Detection in Real Life

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This is a real-world example of intentional error **detection**.

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Another example: The term **Urgent** in the subject line of an email means **this is spam you can ignore.**

# whp means with High Probability

**whp** means **w**ith **h**igh **p**robability. High enough that we will not worry about it not happening.

# Scenario and Conventions

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A **code** is a map  $\{0, 1\}^k$  to  $\{0, 1\}^n$  for error corr. or det.

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We will NOT use Error Detection for McEliece Cipher.

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**Example** Alice wants to send 0110. So she sends 000111111000

What could happen?

1. Bob receives 000111111000, of the right form. Bob is confident he got the msg, and he did.
2. Bob receives 000110111000. 2nd triple is 110. Bob corrects to 111 and is confident he got msg. He did.
3. Bob receives 000110111001. 2nd, 4th triple corrected to 111, 000. He is confident he got msg He did.
4. Bob receives 110110111001. 1st triple corrected to 111. He is confident he got the msg. He did not.

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Alice sends  $b$  by sending

$$b(1, 1, 1) = (b, b, b)$$

$(1, 1, 1)$  is called a **Generating Matrix**. Note that it is  $1 \times 3$ .

## “Bob Sees” with Parity Check Matrices

Alice wants to send  $(b, b, b)$ . There is noise so the msg Bob gets received is  $\vec{b} = (b_1, b_2, b_3)$ . Bob multiplies by matrix  $H$  below.

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} b_1 + b_2 \\ b_1 + b_3 \end{pmatrix}$$

1. If  $b_1 = b_2 = b_3$  then  $H\vec{b} = (0, 0)$ . No errors.
2. If  $b_1 \neq b_2 = b_3$  then  $H\vec{b} = (1, 1)$ . Error in first bit.
3. If  $b_2 \neq b_1 = b_3$  then  $H\vec{b} = (1, 0)$ . Error in second bit.
4. If  $b_3 \neq b_2 = b_1$  then  $H\vec{b} = (0, 1)$ . Error in third bit.

So  $H\vec{b}$  tells Bob if there is an error, and if there is, where it is!

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- ▶ Error Correction: Will catch and correct 1 error.
- ▶ Error Correction: If  $\geq 2$  errors may not catch them.
- ▶ If see  $bG$  then can recover  $b$ . (Trivial but important for later.)

## The (7,4,1) Code Generator Matrix

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$
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Let  $\vec{m} = (m_1, m_2, m_3, m_4)$ .  $\vec{m}G$  is

$$(m_1 + m_3 + m_4, m_1 + m_2 + m_3, m_2 + m_3 + m_4, m_1, m_2, m_3, m_4) = (b_1, b_2, b_3, b_4, b_5, b_6, b_7)$$

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Note that

$$b_1 = b_4 + b_6 + b_7$$

$$b_2 = b_4 + b_5 + b_6$$

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$$\vec{b} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7). \quad H\vec{b} \text{ is} \\ (b_1 + b_4 + b_6 + b_7, b_2 + b_4 + b_5 + b_6, b_3 + b_5 + b_6 + b_7)$$

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- ▶ There are  $7 = 2^3 - 1$  ways that  $h\vec{b} \neq \vec{0}$ . Each one corresponds to which bit is incorrect. (Not obvious.)



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- ▶ This is an error-correcting code with rate  $\frac{4}{7} > \frac{1}{3}$ .
- ▶ This is a (7,4,1)-code.  $|\vec{b}| = 7$ ,  $|\vec{m}| = 4$ , corrects 1 error.

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$$(b_1 + b_4 + b_6 + b_7, b_2 + b_4 + b_5 + b_6, b_3 + b_5 + b_6 + b_7)$$

- ▶ If all coordinates are 0, then no errors.
- ▶ There are  $7 = 2^3 - 1$  ways that  $h\vec{b} \neq \vec{0}$ . Each one corresponds to which bit is incorrect. (Not obvious.)
- ▶ This is an error-correcting code with rate  $\frac{4}{7} > \frac{1}{3}$ .
- ▶ This is a (7,4,1)-code.  $|\vec{b}| = 7$ ,  $|\vec{m}| = 4$ , corrects 1 error.
- ▶ If see  $\vec{m}G$  can recover  $\vec{m}$  easily: the last four bits.

## The (7,4,1) Code or A (7,4,1) Code?

Recall our (7,4,1) Code had a matrix  $G$ , and:

If  $\vec{m} = (m_1, m_2, m_3, m_4)$  then  $\vec{m}G$  is

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So the msg is in slots 4,5,6,7 and the error-correction takes place in slots 1,2,3.

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Is there another  $G$  such that  $\vec{m}G$  is

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Yes.

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Yes. Any rearrangement is a (7,4,1) code.

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Actually we will use matrices much bigger than (7,4,1).

# 1-Error Correcting Codes

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What about 2 errors?

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2 errors and where they are,  $\binom{n}{2}$  possibilities.

# $t$ -Error Correcting Codes

I leave the definition of  $t$ -Error Correcting Codes to you.



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If  $(G, H)$  is an error-correcting code then elements in the image of  $G$  are **codewords**.

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2. McEliece cipher works with any error correcting code; however, in practice they use Goppa codes.
3. We will not have to learn Goppa codes to understand McEliece Cipher.

# Goppa Codes Parameters

We will present parameters for Goppa Codes.

$k$  is length of msg Alice wants to send

$n$  is length of msg Alice sends.

$t$  is how many errors the code can correct. We want this large.

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Here is a table of some known Goppa Code parameters.

$n$	$k$	$t$	$R = k/n$
1024	524	50	0.512
2048	1751	27	0.854
1632	1269	34	0.778

# Back to McEliece Public Key Cryptosystem



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5. Recall: Everything is mod 2.

# An Example of a Perm Matrix

Note that:

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} c \\ a \\ b \\ d \end{pmatrix}$$

The matrix **permutes** the input.

# Perm Matrices

**Def** A **Perm Matrix** is a matrix where

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One can show that

- ▶ If  $P$  is a perm matrix then  $P$  computes a permutation.
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9. Multiply by  $S^{-1}$  to get  $\vec{m}$ .

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1. The usual reason given: Factoring might end up being easy via **Quantum** or **Erika-Guido-Natalya**.
2. We may find that McEliece or LWE might have properties that make it **better** than RSA. This will require using it for a while.

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**CON** RSA, even with the stuff you do to make it really work, seems easier to code up than McEliece. For LWE it's harder to say. Especially if the NSA is listening in.

# My Real World Security Issues

## Email I got recently

Attention: Owner of the Fund, We are delegates of the IMF in conjunction with the assistance of the UN of the AU, the EU and the FBI to pay victims of fraud 3.7 million dollars each. In the course of our investigation, The UN Commission against Crime and the IMF ordered that the money recovered from the scammers be distributed among 10 lucky people around the world. World for compensation. **This email / letter has been sent to you because your email address was found in one of the scam artists' files and the computer is hard drive during our investigation, maybe you were scammed or not, it is being compensated with the sum of us \$3,700,000.** Reconfirm your information as indicated below. 1, Full Names name 2, Contact Address, 3. Nationality, 4. State of origin. Mr Victor Markson

# Article I Read Recently

## Detecting Phishing Attempts

[dl.acm.org/doi/10.1145/3415231](https://dl.acm.org/doi/10.1145/3415231)

**Abstract** To better understand the cognitive process that end users can use to identify phishing msgs, I interviewed 21 IT experts about instances where they successfully identified emails as phishing in their own inboxes. IT experts naturally follow a three-stage process for identifying phishing emails. (1) the email recipient tries to make sense of the email (2) they notice discrepancies: little things that are **off about the email** (3) some feature of the email – usually, the presence of a link requesting an action – triggers them to recognize that phishing is a possible alternative explanation.

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**off about the email** Offering me \$3,700,000 seemed just a little bit off.



## Another Email I Got (excepts)

Urgent - help me distribute my \$12 million to humanitarian aid. This mail might come to you as a surprise and **the temptation to ignore it as unserious could come into your mind but please consider it a divine wish and accept it with a deep sense of humility.**

Since the loss of my husband and also because i had no child to call my own, i have found a new desire to assist the helpless. **I have donated some money to orphans** in Sudan, Ethiopia, Cameroon, Spain, Austria, Germany and some Asian countries. I have **12,000,000.00 u. S. Dollars** which i deposited in a security company in Cotonou Benin Republic that **does not know the real content** to be money and i want you to assist me in claiming the consignment & distributing the money to charity organizations, **i agree to reward you with part of the money** for your assistance, kindness and participation in this godly project. i am in the hospital where i have been undergoing treatment for **oesophageal cancer** and my doctors have told me that i have only a few months to live.

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1. Might fool some people.
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3. It makes it hard to tell who is legit. If I get a letter from a charity I tend to throw it away assuming it is spam.
4. I can't tell the **real Nigerian billionaires** who want to give me \$12,000,000 from the **fake ones**!

# STOP RECORDING LECTURE