BILL, RECORD LECTURE!!!!

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The Shift Cipher

September 1, 2020

Shift Cipher: Encryption, Decryption, Cracking

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- Associate ‘a’ with 0; ‘b’ with 1; . . . ; ‘z’ with 25.
- $s \in \{0, \ldots, 25\}$ (or could think of $s \in \{a, \ldots, z\}$).
- To encrypt using key s, shift every letter of the plaintext by s positions (with wraparound).

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4. Convert numbers to letters to get:
   elooz runvd wdcrr

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He does shift by -3 or can view as shift by $26 - 3 = 23$.

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3. Convert numbers to letters to get: **joshu alike sml**.
4. Figure out spacing to get: **Joshua likes ML**.

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When dealing with mod n we assume the entire universe is $\{0, 1, \ldots, n-1\}$.

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4. Division: Next Slide

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No such x exists.
Fact A number y has an inverse mod 26 if y and 26 have no common factors. Numbers that have an inverse mod 26:

$$\{1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 25\}$$

The Shift Cipher, Formally

- $\mathcal{M} = \{\text{all texts in lowercase English alphabet}\}$
  $\mathcal{M}$ for **Message space**.
  All arithmetic mod 26.
- Choose uniform $s \in \mathcal{K} = \{0, \ldots, 25\}$. $\mathcal{K}$ for **Keyspace**.
- Encode $(m_1 \ldots m_t)$ as $(m_1 + s \ldots m_t + s)$.
- Decode $(c_1 \ldots c_t)$ as $(c_1 - s \ldots c_t - s)$.
- Can verify that correctness holds.

Cracking the Shift Cipher

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Is the Shift Cipher Secure?

- No – only 26 possible keys!
  - Given a ciphertext, try decrypting with every possible key
  - Only one possibility will "make sense"
- Example of a "brute-force" or "exhaustive-search" attack

Example

- Ciphertext `uryyb jbeyq`
- Try every possible key. . .
  - tqxxa iadxp
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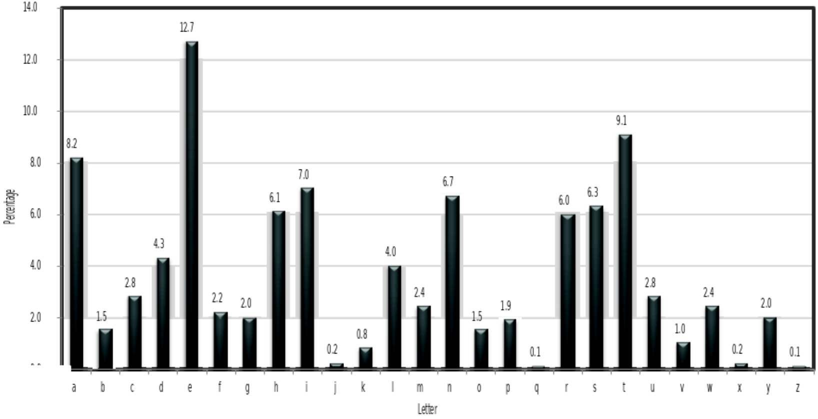
Question: **We** can tell that **hello world** is correct but how can a computer do that. Can we mechanize the process of picking out **the right one**?

Letter Frequencies



Freq Vectors

Let T be a long text. Length N. May or may not be coded.

Let N_a be the number of $a's$ in T.
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The **Freq Vector of T** is

$$\vec{f_T} = \left(\frac{N_a}{N}, \frac{N_b}{N}, \cdots, \frac{N_z}{N} \right)$$

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These are good ideas but do not seem to work.

Vorlons Alphabet: $\{a, b, c, d\}$

- Vorlon freq shifted by 0 is $\vec{f_0} = \{0.5, 0.3, 0.1, 0.1\}$.
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Upshot

$\vec{f_0} \cdot \vec{f_0}$ **big**

For $i \in \{1, 2, 3\}$, $\vec{f_0} \cdot \vec{f_i}$ **small**

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Henceforth $\vec{f_0}$ will be denoted $\vec{f_E}$. E is for *English*

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If 'difficult' cipher used, we may use different IS-ENGLISH function.

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Note: No Near Misses. There will not be two values of s that are both close to 0.065.

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Note: Quite likely to succeed in the first try, or at least very early.