The Vigenère Cipher

September 20, 2020

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MOIRP VUWTC WYDDN BGOFG SDXUU

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Key: $k = (k_1, k_2, \ldots, k_n)$.
Encrypt (all arithmetic is mod 26)

$$Enc(m_1, m_2, \ldots, m_N) =$$

$$m_1 + k_1, m_2 + k_2, \ldots, m_n + k_n,$$

$$m_{n+1} + k_1, m_{n+2} + k_2, \ldots, m_{n+n} + k_n,$$

$$\ldots$$

Decrypt Decryption just reverses the process

Three Kinds of Vigenère Ciphers

The following three slides give three kinds of Vig Ciphers. It is a rough way to divide up types of Vig ciphers. There will be some that are not quite in any category.

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We will be studying this type of Vig cipher today.

VIG TWO: The Book Cipher

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This is called **The Book Cipher**. We will touch on it briefly in a later lecture (or today, we'll see how far we get).

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It is usually done with alphabet $\{0, 1\}$ or $\{0, \ldots, 9\}$, not $\{a, \ldots, z\}$.

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- Might not have even been secure then...
- History of Cryptography is hard since, unlike most science, people can discover things and NOT brag about it.

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Important: Very likely that **aiq** encrypted **the same** 3-letter sequence and hence the **length** of the key is a divisor of
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The only possible L's are 1,3,5,15.

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Good Enough: We got the key length down to a small finite set.

Important Point About Letter Freq

Assume (it's roughly true): In an English text T of length N:

e occurs $\sim 13\%$ t occurs $\sim 9\%$ a occurs $\sim 8\%$

Etc- other letters have frequencies that are true for all texts.

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Assume (it's roughly true): In an English text T of length N, if $i \ll N$, then if you take **every ith letter of T**:

e occurs $\sim$ 13% t occurs $\sim$ 9% a occurs $\sim$ 8%

Etc- have the other letters same frequencies as normal texts.

Variant on Is-English (I)

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- If T is shifted then $f_E \cdot f_T \sim\leq 0.035$.

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New Observation $f_T \cdot f_T \sim 0.065$.

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Our question T is ciphertext coded with Vig Cipher. Eve thinks the key length is L. Let S be **every Lth letter of T**. SO

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- If keylength is not L then S is a ... a real mess!! $f_S \cdot f_S$ will be small.

Upshot We have a test whether some text is from the shift-cipher or not. We will use it on the every-Lth-letter text of T.

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- Just to make sure, check another stream.

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Note: With modern computers use Method TWO. In the pre-computation era Method ONE was used.

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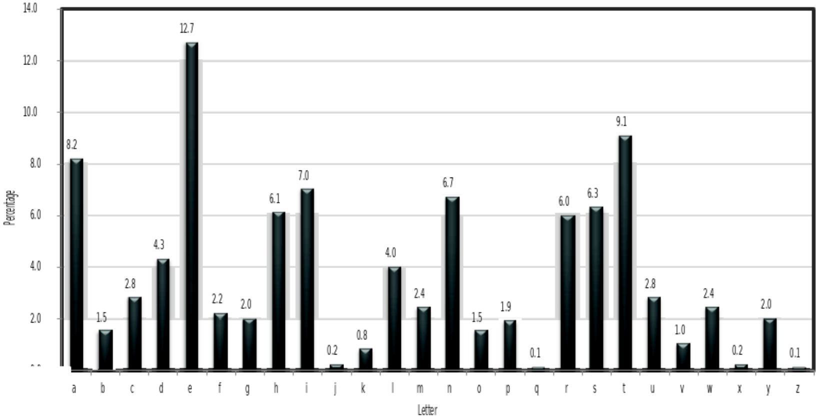
3. You now know all shifts for all positions. Decrypt!

Using Plaintext Letter Frequencies



Making Vig Harder to Crack

Usual Vig

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Jacob Prinz is a Physics Major

encrypts to

MOIRP VUWTC WYDDN BOFGS DXUU

Getting More Out of Your Phrase

If the key was

Corn Flake

You would get a key of length 9. We want **More**.

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If the key was

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Corn is 4 letters long. **Flake** is 5 letters long.
We form a key of length $LCM(4,5) = 20$. (Won't fit on line! Oh Well.)

C	O	R	N	C	O	R	N	C	O	R	N	C	O	R	N	C
F	L	A	K	E	F	L	A	K	E	F	L	A	K	E	F	L
7	25	17	23	6	19	2	13	12	18	22	24	2	24	21	18	1

ADD it up to get new 20-long key.

Getting More Out of Your Phrase (cont)

C	O	R	N	C	O	R	N	C	O	R	N	C	O	R	N	C
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This new key has two advantages:

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1. Longer Key for Eve to Crack, but not harder for Alice and Bob to transmit.

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This new key has two advantages:

1. Longer Key for Eve to Crack, but not harder for Alice and Bob to transmit.
2. The key is not an English Phrase, so harder for Eve.

Getting More Out of Your Phrase (cont again)

If phrase is **Wheel of Fortune** and you did the above trick, how long a key do you get? **Discuss**

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$LCM(5, 2, 7) = 70$.

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In an older era the LCM trick may have made Vig go from crackable to uncrackable.

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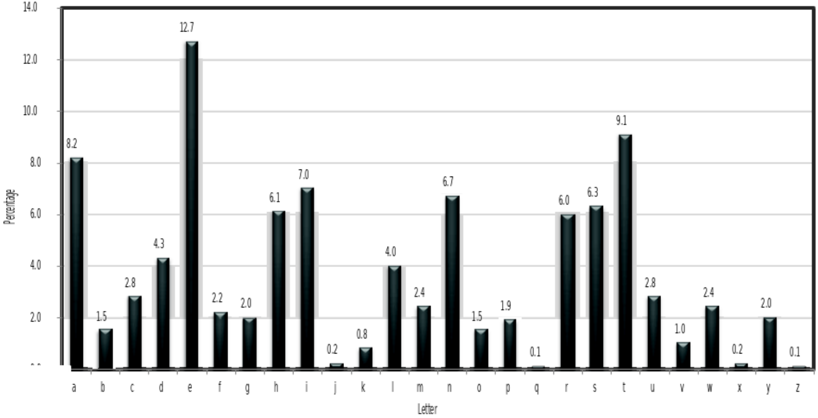
1. Before modern computer era: YES.
2. Now. NO.

How to Crack the Vig Book Cipher

Key: Both Key and Text have the English Lang Frequencies.



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Eve knows that (First Letter in Key) + (First Letter in Text) = 3. Hence the following are the only possibilities for (Letter in Key, Letter in Text) are:

$(a, d), (z, e), (y, f), (w, g), \ldots, (b, c)$

Only 26 possibilities. What of it? **Discuss**

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Only 26 possibilities. What of it? **Discuss**
Some of the pairs are more likely than others.

1. **Both** the key **and** the text are in English.
2. (z, e): Hmm, z is unlikely but e is likely.
3. (a, d): Hmm, seems more likely than (z, e).
4. Can rank which are more likely (e.g., add or mult the freqs).
5. Can then use adjacent letters and freq of adjacent pairs, and rank them.
6. Triples. Etc.

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Were these good choices? NO.

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Were these good choices? NO. They are books Eve might guess.

Bill Should Not Use. . .

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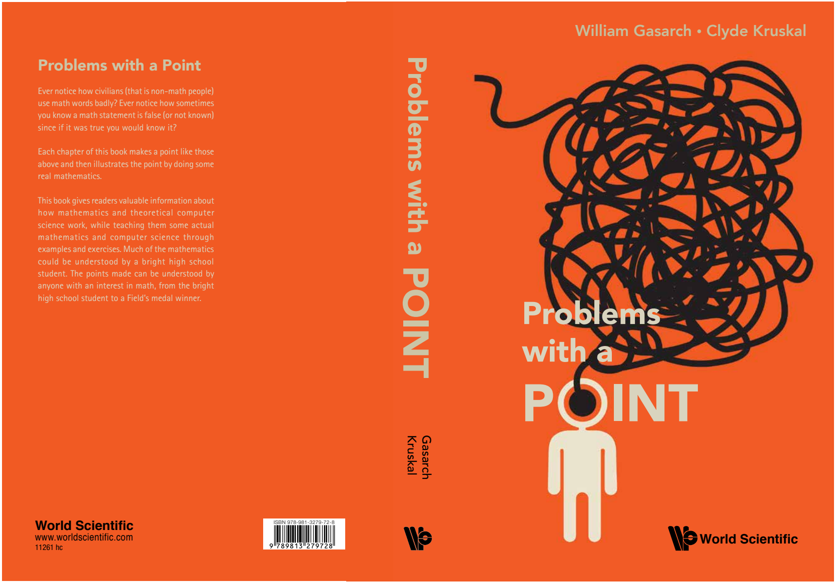

Problems with a Point

Ever notice how civilians (that is non-math people) use math words badly? Ever notice how sometimes you know a math statement is false (or not known) since if it was true you would know it?

Each chapter of this book makes a point like those above and then illustrates the point by doing some real mathematics.

This book gives readers valuable information about how mathematics and theoretical computer science work, while teaching them some actual mathematics and computer science through examples and exercises. Much of the mathematics could be understood by a bright high school student. The points made can be understood by anyone with an interest in math, from the bright high school student to a Field's medal winner.

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