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3. To send b Alice sends $(\vec{r}; D)$ where $D \equiv C + e + \frac{bp}{4}$.
4. Bob computes $\vec{r} \cdot \vec{k} \equiv C$. If $D \sim C$, $b = 0$, else $b = 1$.

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  How can Bob use the noisy equations to encode a bit?

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We add lots of equations, so γ **very** small.

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Note Any sum of the eqs also has $(1, 10, 21, 89)$ as "answer."

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   Note $\vec{k}$ satisfies the noisy equations and any sum of them.
3. Bob wants to send bit b. He picks a uniform random set of the public noisy equations and adds them, AND adds $\frac{bp}{2}$.

$$s_1 x_1 + \cdots + s_n x_n \sim D' + \frac{bp}{2} \text{ iff } b = 0$$

   D' is sum of Ds. Broadcasts $(\vec{s}; F)$ where $F = D' + \frac{bp}{2}$.

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Details omitted, but:

- Will need to take $\gamma \leq \frac{p}{2m}$.
- Will need p large so that $\frac{p}{2m}$ is large enough for a variety of error values for increased security.

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Hard? We discuss why this problem is thought to be hard.

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Theorem If Eve can crack the LWE-public cipher then Eve can solve the LWE-problem. Note that this is the direction you want.

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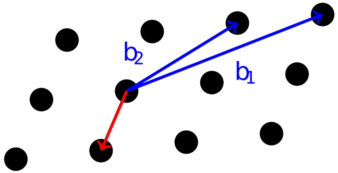
We now go into that some more.

Shortest Vector Problem (SVP)

SVP Given a lattice, find the shortest Vector out of the origin.



(Picture by Sebastian Schmittner - Own work, CC BY-SA 4.0,
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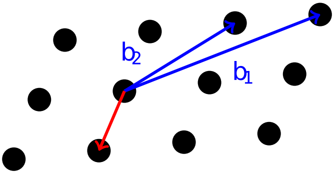

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Hardness Known to be NP-hard under randomized reductions.

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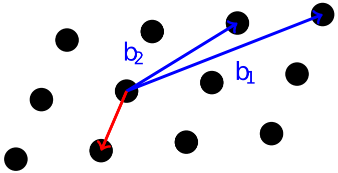

(Picture by Sebastian Schmittner - Own work, CC BY-SA 4.0, `https://commons.wikimedia.org/w/index.php?curid=44488873`)

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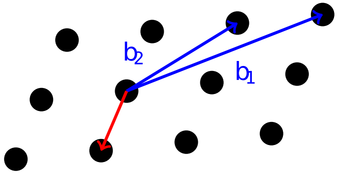

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Caveat Regev showed the quantum reduction in 2009. Peikert obtained a randomized reduction in 2014. The quantum reduction works for a wider range of parameters.

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Note that what I showed here were the IDEAS behind LWE-public. Getting it to actually work requires many modifications.

BILL, STOP RECORDING LECTURE!!!!

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