BILL, RECORD LECTURE!!!!

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The Shift Cipher

Shift Cipher: Encryption, Decryption, Cracking

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- $s \in \{0, \ldots, 25\}$ (or could think of $s \in \{a, \ldots, z\}$).
- To encrypt using key s, shift every letter of the plaintext by s positions (with wraparound).

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4. Convert numbers to letters to get:
   elooz runvd wdcrr

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He does shift by -3 or can view as shift by $26 - 3 = 23$.

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3. Convert numbers to letters to get: **joshu alike sml**.
4. Figure out spacing to get: **Joshua likes ML**.

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When dealing with mod n we assume the entire universe is $\{0, 1, \ldots, n-1\}$.

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4. Division: Next Slide

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Fact A number y has an inverse mod 26 if y and 26 have no common factors. Numbers that have an inverse mod 26:

$$\{1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 25\}$$

Proof is not hard but I won't be doing it **Proof is not hard but I won't be doing it**

The Shift Cipher, Formally

- $\mathcal{M} = \{\text{all texts in lowercase English alphabet}\}$
  $\mathcal{M}$ for **Message space**.
  All arithmetic mod 26.

- Choose uniform $s \in \mathcal{K} = \{0, \ldots, 25\}$. $\mathcal{K}$ for **Keyspace**.

- Encode $(m_1 \ldots m_t)$ as $(m_1 + s \ldots m_t + s)$.

- Decode $(c_1 \ldots c_t)$ as $(c_1 - s \ldots c_t - s)$.

- Can verify that correctness holds.

Cracking the Shift Cipher

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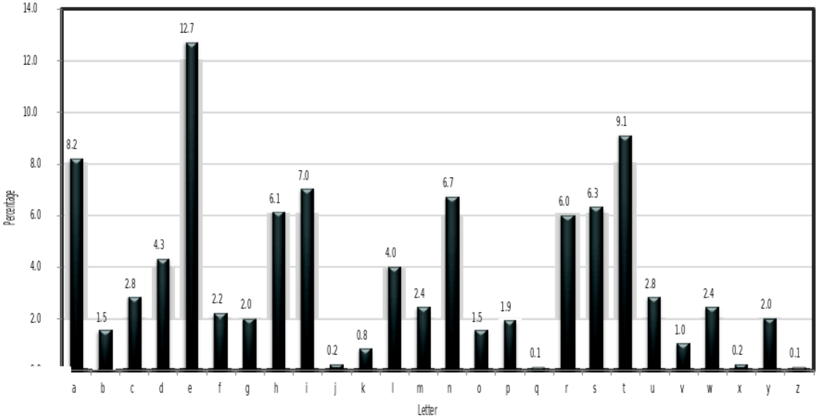
We want a way for **a program** to tell us if a text **looks like English**.

Letter Frequencies



Freq Vectors

Let T be a long text. Length N. May or may not be coded.

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The **Freq Vector of T** is

$$\vec{f_T} = \left(\frac{N_a}{N}, \frac{N_b}{N}, \cdots, \frac{N_z}{N} \right)$$

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These are good ideas but do not seem to work.

Vorlons Alphabet: $\{a, b, c, d\}$

- Vorlon freq shifted by 0 is $\vec{f_0} = \{0.5, 0.3, 0.1, 0.1\}$.
- Vorlon freq shifted by 1 is $\vec{f_1} = \{0.1, 0.5, 0.3, 0.1\}$.
- Vorlon freq shifted by 2 is $\vec{f_2} = \{0.1, 0.1, 0.5, 0.3\}$.
- Vorlon freq shifted by 3 is $\vec{f_3} = \{0.3, 0.1, 0.1, 0.5\}$.

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$$\vec{f_0} \cdot \vec{f_0} = 0.5^2 + 0.3^2 + 0.1^2 + 0.1^2 = 0.36$$

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Upshot

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Henceforth $\vec{f_0}$ will be denoted $\vec{f_E}$. E is for *English*

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If 'difficult' cipher used, we may use different IS-ENGLISH function.

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The numbers (0.065,0.038) are **not mathematical** and are the empirical parameters for English. Different languages will have different parameters, but all will have a large gap between shifted and non-shifted.

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Note: Only one value of s will cause **Is English**(T_s) ~ 0.065

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Note Quite likely to succeed in the first try, or at least very early.
Why Would it Not Succeed on First Try? Short Text, strange text, or the person encoding does not like the letter **e**.