BILL, RECORD LECTURE!!!!

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IF YOU DIDN"T GET THE EMAIL

LET ME KNOW.
I send an email to the class on Aug 31 at night asking you to respond. If you DID NOT get that email then see me TODAY after class so I can get the email you want me to use and add you to the list.

GRADESCOPE

Gradescope Code: P56D84

The Shift Cipher (cont)

A Caveat on Cracking The Shift Cipher

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For more complicated ciphers we may need more sophisticated IS-ENGLISH programs and the parameters may be harder to fine-tune.

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Is this a good approach? No– we spend 26 steps on every letter!

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This is much faster.

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Discuss

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But if we have a few candidates for IS-ENGLISH there may be other ways to pick out the real one.

Variants of the Shift Cipher

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Include other symbols depending on the branch of math. E.g., $\wedge, \vee$ for logic.

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What to do? Find distribution of alphabet for these types of docs. Write code sim to **Is-English** and try all shifts.

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4. Parity Checks.

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3. Needed to use **IS-ENGLISH** program which we will use later as well.

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Arguments Against:

- When initially use a cipher then Eve won't know what the cipher is for a while (months? days? hours?) For that (perhaps short) period of time the secrecy of the cipher will make it hard to crack.

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Since the answer to both questions was **the same**, namely **A**,
Eve knows Saj is working for either **both** or **neither.**

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Does this leak information Discuss.

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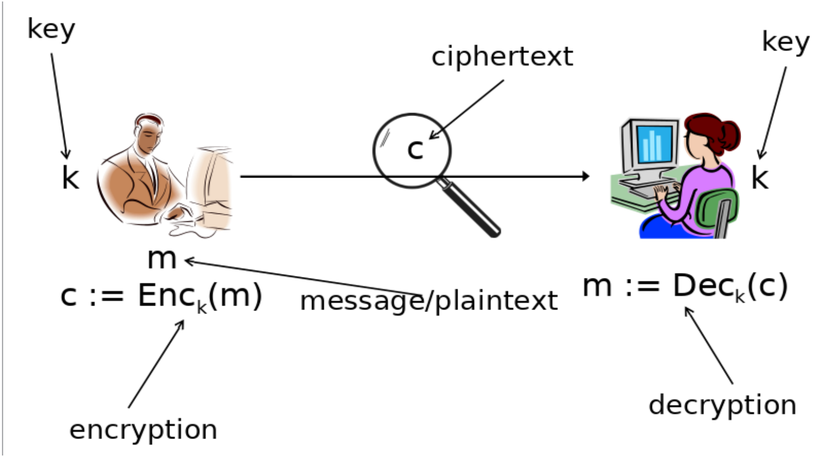
For Now A lesson in how even defining **security** and **leak** must be done carefully.

Private-Key Encryption



key

ciphertext

key

k

c

k

m

$c := \mathrm{Enc}_k(m)$

message/plaintext

$m := \mathrm{Dec}_k(c)$

decryption

encryption

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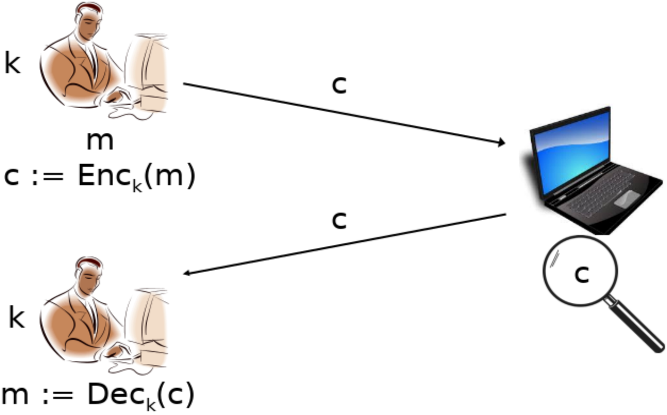

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Private-key encryption

- A *private-key encryption scheme* is defined by a message space $\mathcal{M}$ and algorithms **(Gen, Enc, Dec)**
  - **Gen** (key generation algorithm): outputs $k \in \mathcal{K}$ (For SHIFT this is $k \in \{0, \dots, 25\}$. Should 0 be included?)
  - **Enc** (encryption algorithm): takes key k and message $m \in \mathcal{M}$ as input; outputs ciphertext c

    $$c \leftarrow Enc_k(m)$$

    (For SHIFT this is $\text{Enc}(m_1, \dots, m_n) = (m_1 + k, \dots, m_n + k)$.)
  - **Dec** (decryption algorithm): takes key k and ciphertext c as input; outputs m or "error"

    $$m := Dec_k(c)$$

    (For SHIFT this is $\text{Dec}(c_1, \dots, c_n) = (c_1 - k, \dots, c_n - k)$.)

    $$\forall k \text{ output by Gen } \forall m \in \mathcal{M}, Dec_k(Enc_k(m)) = m$$

(For SHIFT this is $(m + k) - k = m$)

BILL, STOP RECORDING LECTURE!!!!

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