

# BILL RECORD THIS LECTURE

# Gen Sub Cipher and Random-Looking Ciphers

# General Substitution Cipher

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- ▶ Shift and Affine both use some math—hence math can be used against them.

We present the **General Substitution Cipher** which:

- ▶ Has a large keyspace.
- ▶ Does not use any math.

# General Substitution Cipher

**Def Gen Sub Cipher** with perm  $f$  on  $\{0, \dots, 25\}$ .

1. Encrypt via  $x \rightarrow f(x)$ .
2. Decrypt via  $x \rightarrow f^{-1}(x)$ .

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If the message is **FBI** it will encrypt to **GIH**.

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**Yes** Eve can use Freq Analysis

# Freq Analysis

Alice sends Bob a LONG text encrypted by Gen Sub Cipher.  
Eve finds freq of letters, pairs, triples, . . . .

Text in English.

1. Can use known freq: *e* is most common letter, *th* is most common pair.
2. Depending on topic may need to adjust frequencies. For example, if message is about the Mid East then *q* is more common (Iraq, Qatar).

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That should have been the ad slogan for watching Jeopardy.  
And now it can't be :-)

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**We assume long normal texts!**

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David Zhen has a program that cracks the gen sub cipher.

# Random-Looking Ciphers

# Alternatives to Gen Sub (History)

**In the Year 2020** Alice can easily generate a **random** permutation of  $\{a, \dots, z\}$  and send it to Bob. Key length is not a problem.

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1. We show one such methods.
2. These methods are primitive examples of **psuedo-random generators** which take a short string and make a **random-looking** much longer string. These are important in crypto. We will encounter them again.

## Keyword-Shift Cipher. Key is (Phrase,Shift)

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2. Now do Shift 4 on this:

f	g	h	i	j	a	c	k	b	d	e
---	---	---	---	---	---	---	---	---	---	---

This is where  $a, b, c, \dots$  go, so:

a	b	c	d	e	f	g	h	i	j	k
f	g	h	i	j	a	c	k	b	d	e

# Keyword-Shift Cipher. Key is (Phrase,Shift) (cont)

To encrypt use:

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>
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To decrypt you invert the table:

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>
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# From Short Key Got Rand-Looking Perm(?)

From (jack,4) (which is short) we got

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5. With 4-let keywords, not that rand looking.

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I leave the rest to you. Find the encode and decode tables and see  
if they **look random**.