HW 03 Some Solutions

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Problem 3

Theorem Let X be a countable infinite set of points in the plane, no three colinear. Then there exists $Y \subseteq X$, $|Y| = \infty$, such that all of the triangles formed have different areas.
Proof Order the points in X arbitrarily, so $X = \{p_1, p_2, p_3, \ldots\}$.
$\mathrm{COL}\colon \binom{N}{3} \to R$ via $\mathrm{COL}(i, j, k)$ area $\mathrm{TRI}(p_i, p_j, p_k)$
By Can Ramsey ($\exists H \subseteq \mathrm{N}$, $|H| = \infty$, H A-homog, some $A \subseteq \{1, 2, 3\}$.

$$Y = \{p_i : i \in H\}.$$

We show that the only A-homog set possible is $\{1, 2, 3\}$-homog (Rainbow).

If $\{1,2\}$-Homog then Contradiction

If H is $\{1,2\}$-homog then every triangle that has p_1, p_2 has same area.
AREA of the following triangles is the same:
$TRI(p_1, p_2, p_3)$
$TRI(p_1, p_2, p_4)$
$TRI(p_1, p_2, p_5)$
$TRI(p_1, p_2, p_6)$
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Three of them are on the same side of that line.

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Similar for $\{1,3\}$-Homog, $\{2,3\}$-Homog.

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Proof is the same as last case.
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If $\emptyset$-Homog

If H is $\emptyset$-homog then every triangle that has the same area. Proof the same as the other two cases.

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- If $X \subseteq R^2$ then there exists infinite subset $Y \subseteq X$ such that all of the distances between points in Y are different.
- If $X \subseteq R^2$, no three colinear, then there exists infinite subset $Y \subseteq X$ such that all of the areas of triangles formed by three points of Y are different.

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VOTE

- These are Applications!
- These are "Applications"
- These are crap!

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Write a program that randomly color the edges of K_5 by coloring RED with prob p and BLUE with prob $1 - p$ and count the number of mono triangles. From this make conjectures.

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Conjecture For all $\mathrm{COL}\colon \binom{[5]}{2} \rightarrow [2]$ there are **never** 6 or 8 or 9 mono triangles.

This is actually **True**.

Problem 5 (Extra Credit)

Prove the 3-ary Can Ramsey.
Will do on the White Board.