Homework 04

Morally Due Tue Feb 22 at 3:30PM. Dead Cat Feb 24 at 3:30

1. (0 points) What is your name? Write it clearly. When is the take-home midterm due?

2. (40 points) Assume $(X, \preceq_X)$ and $(Y, \preceq_Y)$ are wqo. Consider the ordering $(X \times Y, \preceq)$ where $\preceq$ is defined as

$$(x_1, y_1) \preceq (x_2, y_2) \text{ iff } x_1 \preceq_X x_2 \text{ AND } y_1 \preceq_Y y_2.$$

Show that $(X \times Y, \preceq)$ is a wqo.

GO TO NEXT PAGE

3. (50 points) Assume $(X, \preceq)$ is a wqo. Let $P^{finite}(X)$ be the set of finite subsets of X. Let $\preceq'$ be the following order on $P^{finite}(X)$.

   Let $Y, Z \in P^{finite}(X)$.

   $Y \preceq' Z$ iff

   there exists a FUNCTION $f : Y \rightarrow Z$ such that $(\forall y \in Y)[y \preceq f(y)]$.

   (a) (20 points) Prove or disprove: $(P^{finite}(X), \preceq')$ is a wqo.

   (b) (15 points) Modify $\preceq'$ such that the function f has to be injective (also called 1-1). Prove or disprove: $(P^{finite}(X), \preceq')$ is a wqo.

   (c) (15 points) Modify $\preceq'$ such that the function f has to be surjective (also called onto). Prove or disprove: $(P^{finite}(X), \preceq')$ is a wqo.

4. (10 points) GOTO my webpage of funny music and GOTO the section on Math Songs

   `https://www.cs.umd.edu/~gasarch/FUN//funnysongs.html`

   - Listen to the Bolzano Weirstrauss rap- or as much of it as you can stand. Comment on it.
   - Pick ANY OTHER math song AT RANDOM and listen to it. Is it better than the BW rap (hint: YES). Comment on it.

GO TO NEXT PAGE

5. (Extra Credit- NOT towards your grade but towards a letter I may one day write for you) (This will look like a prior extra credit but it's a new problem.)

   Definition A *bipartite* graph is a graph with vertices $A \cup B$ and the only edges are between vertices of A and vertices of B. A and B can be the same set. We denote a bipartite graph with a 3-tuple (A, B, E).

   Notation $K_{n,m}$ is the bipartite graph $([n], [m], [n] \times [m])$.

   Notation $K_{\mathsf{N},\mathsf{N}}$ is the bipartite graph $(\mathsf{N}, \mathsf{N}, \mathsf{N} \times \mathsf{N})$.

   Definition If COL is a c-coloring of the edges of $K_{\mathsf{N},\mathsf{N}}$ then (H_1, H_2) is a homog set if c restricted to $H_1 \times H_2$ takes on only 1 value (I changed the wording on this so I can generalize it later.)

   RECALL In a prior extra credit problem we DISPROVED the following:

   For every 2-coloring of the edges of $K_{\mathsf{N},\mathsf{N}}$ there exists H_1, H_2 infinite such that (H_1, H_2) is a homog set.

   In other words we showed the following:

   There IS a 2-coloring of the edges of $K_{\mathsf{N},\mathsf{N}}$ such that there is NO H_1, H_2 infinite such that (H_1, H_2) is a homog set.

   This inspires the following definition.

   Definition Let $d \le c$. If COL is a c-coloring of the edges of $K_{\mathsf{N},\mathsf{N}}$ then (H_1, H_2) is a d-homog set if c restricted to $H_1 \times H_2$ COL takes on $\le d$ values.

   SO to recap- we could have a 2-coloring of the edges of $K_{\mathsf{N},\mathsf{N}}$ where there is no 1-homog set. But there is clearly a 2-homog set, namely (N, N).

   And now FINALLY the problem:

   For ever $k \ge 3$ Prove or disprove: For every k-coloring of the edges of $K_{\mathsf{N},\mathsf{N}}$ there exists H_1, H_2 infinite such that (H_1, H_2) is a 2-homog set.