Homework 07

Morally Due Tue March 29 at 3:30PM. Dead Cat March 31 at 3:30

IN THIS HW WHENEVER I SAY "A SET OF POINTS IN THE PLANE" I MEAN THAT THEY HAVE NO THREE COLINEAR.

1. (0 points) What is your name? Write it clearly. When is the take-home final due?

2. (35 points) Let $N(k)$ be the least n such that for all sets of n points there is a subset of k of them that form a convex k-gon.

   We begin a proof that $N(k)$ exists and you need to finish it.

   We show that $n = R_3(k)$ suffice. Let X be a set of $n = R_3(k)$ points in the plane. Let the points be $p_1, p_2, \ldots, p_n$.

   Color (p_i, p_j, p_j) (with $i < j < k$) RED if p_i, p_j, p_k is clockwise.

   Color (p_i, p_j, p_j) (with $i < j < k$) BLUE if p_i, p_j, p_k is counter clockwise.

   The Homogenous set of size k form a convex k-gon because FILL THIS IN.

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3. (35 points)

Def JULY(k) is the least $n \geq$ such that for all 2-colorings of $\binom{\{k,\ldots,n\}}{2}$ there exists a set H such that

- $|H| \geq 3$,
- $|H| > MIN(H)$,
- COL restricted to $\binom{H}{2}$ is constant.

Find a number A such that you can prove JULY$(1) \leq A$.

(I have a proof with $A = 8$ but given that the original version of this problem was incorrect, I am phrasing it this way so it can't go wrong.)

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4. (30 points) Recall:

If $n \equiv 1 \pmod{2}$ *then for any* $\mathrm{COL}\colon \binom{[n]}{2} \rightarrow [2]$ *there exists at least*

$$\frac{n^3}{24} - \frac{n^2}{4} + \frac{5n}{24}$$

monochromatic K_3*'s.*

We will vary this in two ways.

(a) (15 points) Find a function f such that the followings is true:
If $n \equiv 0 \pmod{2}$ *then for any* $\mathrm{COL}\colon \binom{[n]}{2} \rightarrow [2]$ *there exists at least* $f(n)$ *monochromatic* K_3*'s.*
Prove your result.

(b) (15 points) We are interested in what happens if you have THREE colors. Do some empirical studies to try to find a function f such that the following holds:
If $\mathrm{COL}\colon \binom{[n]}{2} \rightarrow [3]$ *then there exists at least* $f(n)$ *monochromatic* K_3*'s. (*$f(n)$ *an be approximate. For example, if the problem was for 2-coloring then* $f(n)$ *could be* $\frac{n^3}{24}$*.)*
(HINT: Use the code you wrote for the midterm; however, only use the case of $p_1 = p_2 = p_3 = \frac{1}{3}$.)

(c) (Extra Credit, 0 points) PROVE a result along the lines of:
If n *satisfies condition YOU FILL IN and* $\mathrm{COL}\colon \binom{[n]}{2} \rightarrow [3]$ *then there exists at least* $f(n)$ *monochromatic* K_3*'s.)*
(HINT: Use the code you wrote for the midterm; however, only use the case of $p_1 = p_2 = p_3 = \frac{1}{3}$*.)*

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5. (Extra Credit, but THINK ABOUT IT. WARNING- I have not done this problem)

   Let X be an infinite set of points $p_1, p_2, p_3, \ldots$. Let $\mathrm{COL}\binom{\mathsf{N}}{3} \rightarrow \omega$ *be defined as follows:*

   $$\mathrm{COL}(i,j,k) = \textit{ the number of points inside the } (i,j,k) \textit{ triangle.}$$

   Apply the 3-ary Can Ramsey Theorem to this Coloring. NOW WHAT?

6. (Extra Credit, but THINK ABOUT IT–WARNING: the way I know how to do this is based on material you have not seen) We want to write a sentence ϕ in the language of graphs such that

   $G \models \phi$ IFF G has an even number of vertices.

   Is this possible?