Logic Seminar Computability Cheat Sheet

Notation 0.1

1. $M_0, M_1, \ldots$ is a standard list of Turing Machines (TMs).

2. $M_{e,s}(x)$ means that we run M_e for s steps. $M_e(x)\downarrow$ means that M_e halt on input x.

3. $W_e = \{x\colon M_e(x)\downarrow\}$. $W_{e,s} = \{x\colon M_{e,s}(x)\downarrow\}$.

Sets are classified in the Arithmetic hierarchy.

Notation 0.2

1. $A \in \Sigma_0$ if A is computable. $A \in \Pi_0$ if A is computable.

2. For $i \ge 1$ $A \in \Sigma_i$ is there exists $B \in \Pi_{i-1}$ such that $A = \{x \mid (\exists y)[(x,y) \in B]\}$

3. For $i \ge 1$ $A \in \Pi_i$ is there exists $B \in \Sigma_{i-1}$ such that $A = \{x \mid (\forall y)[(x,y) \in B]\}$

Examples and Facts

1. $W_0, W_1, \ldots$ is a list of all Σ_1 sets.

2. $\Sigma_0 \subset \Sigma_1 \subset \Sigma_2 \subset \cdots$. AND $\Pi_0 \subset \Pi_1 \subset \Pi_2 \subset \cdots$.

3. If $A = \{x\colon (\exists y)[B(x,y)]$ where $B \le_{\mathrm{T}} HALT$ then $A \in \Sigma_2$.

Theorem 0.3 *There is a computable $COL\colon \binom{N}{2} \rightarrow [2]$ such that there is no infinite Σ_1 homog set.*

Theorem 0.4 *There is a computable $\mathrm{COL}\colon \binom{N}{2} \rightarrow [2]$ such that there is no infinite Σ_2 homog set.*

Theorem 0.5 *For every computable coloring $\mathrm{COL}\colon \binom{N}{2} \rightarrow [2]$ there is an infinite Π_2 homog set.*