Topics Not Covered in Grad Ramsey 2020

Exposition by William Gasarch

May 18, 2020

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- Some combination of the above.

Could Have Covered: VDW

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Poly VDW Thm

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Recall **VDW Thm** For all k, c there exists $W = W(k, c)$ such that for all $\mathrm{COL}\colon [W] \to [c]$ there exists a, d

$$a, a+d, \ldots, a+(k-1)d \text{ all the same color}$$

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Poly VDW Thm For all $p_1, \ldots, p_k \in \mathbb{Z}[x]$ such that $p_i(0) = 0$ for all $i \in [k]$, and $c \in \mathbb{N}$, there exists $W = W(p_1, \ldots, p_k; c)$ such that for all $\mathrm{COL}\colon [W] \to [c]$ there exists a, d

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Walters gave elementary proof which I would have presented.

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4. **Research** Get better bounds for poly VDW numbers, perhaps by programming. Maybe use a SAT solvers. Or write one geared to this purpose.

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Can VDW For all k there exists $W = W(k)$ such that for any $\mathrm{COL}\colon [W] \rightarrow [\omega]$ there exists a, d such that either

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Research Better bounds on Can VDW Numbers.

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Research The proof gives VDW-like bounds. Hard NT gives better bounds. Get better bounds in elementary way.

Folkman's Thm

Rado's Thm Let $a_1, \ldots, a_k \in \mathbb{Z}$. TFAE

- Some subset of the a_i's sums to 0.
- For all c, for all $\mathrm{COL}\colon \mathbb{N} \rightarrow [c]$ there exists mono solution to

$$a_1x_1 + \cdots + a_kx_k = 0.$$

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Great thm, nice proof. Might cover it in the future.

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  Can ask this question for many equations.
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- Caution: Some of this may be known.

Hilbert's Cube Lemma

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  This is Roth's proof done with the ideas showing and the computation rightly put into the background.
- **Research** Get better bounds: How big a subset of $\{1, \ldots, 1000\}$ before guaranteed a 3-AP? 4-AP? etc.

A Stupid App of Schur's Thm to Number Theory

Schur's Theorem is a special case or Rado's Theorem.

Schur's Thm For all c there exists $S = S(c)$ such that for all $\mathrm{COL}\colon [S] \rightarrow [c]$ there exists x, y, z same color such that $x + y = z$.

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Gasarch proved:
Thm (Schur's Thm + FLT(4) implies there are an infinite number of primes. `https://www.cs.umd.edu/users/gasarch/COURSES/858/S20/notes/schurflt.pdf`

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- The general idea of the Rec or Rev math program on a lower level I have been told is intriguing.

Rado's Theorem over the Reals

Vote

For all $COL : \mathbb{R} \rightarrow \mathbb{N}$ there exists w, x, y, z all the same color:

$$w + x = y + z$$

- TRUE
- FALSE
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Sample Thm

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- Their are many results and the proofs are elementary.

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$$R(C_k) = \begin{cases} 6 & \text{if } k = 3 \text{ or } k = 4 \\ 2k-1 & \text{if } k \geq 5 \text{ and } k \equiv 1 \pmod 2 \\ \frac{3k}{2} - 1 & \text{if } k \geq 4 \text{ and } k \equiv 0 \pmod 2 \end{cases} \quad (1)$$

- Their are many results and the proofs are elementary.
- I would need to learn it (this is a PRO). I may have a student writeup the proofs for a project. Then I'll see if its interesting.

Other Ramsey Numbers

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- For every result of this type see `https://www.combinatorics.org/files/Surveys/ds1/ds1v15-2017.pdf`

Research Projects

- Actually FIND the colorings.
- Simplify or unify the proofs
- **Ramsey Games** Example: Parameter k, n. Players RED and BLUE alternate coloring the edges of K_n. RED goes first. The first player to get a C_k in their color wins.
    1. For which n does RED have a winning strategy?
    2. Design an ML to play this well (my REU project)
    3. EVERY thm in Ramsey Thm (and the VDW part) can be made into a game and lead to research projects.

Better Bounds on 3-Hypergraph Ramsey

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Research Use their technique on other Ramsey problems.

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- Do we really need more Can Ramsey in the course?

Large Can Ramsey

The following is well known; however, I may be the first person to write down the proof.

http://www.cs.umd.edu/~gasarch/COURSES/858/S20/notes/canlarge.pdf

Thm For all k there exists $n = n(k)$ such that for all $\mathrm{COL}\colon \binom{\{k,\ldots,n\}}{2} \to [\omega]$ there is a large set that is either homog, min-homog, max-homog, rainbow.

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Research Get the bound in terms of LR_3 or lower.

a-ary Can Ramsey

Thm For all $a, k \in \mathbb{N}$ there exist $C = C(a,k)$ such that for all $\mathrm{COL}\colon [\binom{[C]}{a}] \to [\omega]$ there exists a set H, $|H| = k$ and $1 \le i_1 < \cdots < i_L \le a$ such that for all $p_1 < \cdots < p_a \in H$ and $q_1 < \cdots < q_a \in H$

$$\mathrm{COL}(p_1, \ldots, p_a) = \mathrm{COL}(q_1, \ldots, q_a) \text{ iff } (p_{i_1}, \ldots, p_{i_L}) = (q_{i_1}, \ldots, q_{i_L})$$

- Similar to the proof on graphs, but messier.
- *On canonical Ramsey numbers for coloring three-element sets* by Lefmann and Rodl behind paywalls, lost to humanity.
- Optimal results due to Shelah: `https://arxiv.org/abs/math/9509229` A hard read.

Research Give easier proofs of bounds.

Could have Covered: Euclidean Ramsey Theory

Exposition by William Gasarch

May 18, 2020

Euclidean Ramsey Theory

Sample Thm Let T be a triangle with a 30, 90, or 150 degree angle. For every 2-coloring of $\mathbb{R}^2$ there exists three points that form triangle T (note- actually form T, not just similar to T) that are monochromatic.

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- For more: `https://www.csun.edu/~ctoth/Handbook/chap11.pdf`

Results Bill Likes But Would be Hard to Teach:VDW

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App of 3-Free Sets to Complexity Theory

Def L is a language. Game:

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- Alice is Poly time and she has x, $|x| = n$.

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Def L is a language. Game:

- Alice is Poly time and she has x, $|x| = n$.
- Bob is all powerful and he has nothing.
- They cooperate to determine if $x \in L$. Alice could just send Bob x. That takes n bits.

App of 3-Free Sets to Complexity Theory Cont

Let L be the set of all 3-colorable graphs (or any NPC graph problem). Note size is $O(n^2)$. Is there a protocol for Alice and Bob in $O(n^{2-\epsilon})$ bits for some $\epsilon > 0$?

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- **Research** Come up with an elementary proof.

Results Bill Likes But Would be Hard to Teach:Ramsey

Exposition by William Gasarch

May 18, 2020

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- Could teach this thm next time.
- **Research** Look at col G to get mono H for other G and H.

Results Bill Likes But Would be Hard to Teach: Complexity

Exposition by William Gasarch

May 18, 2020

Complexity: Π_2^p Completeness of Arrow

Def $G \to (H_1, H_2)$ means that for every 2-coloring of the edges of G there is either a **RED** H_1 or a **BLUE** H_2.

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Marcus Schaefer proved the following.
Thm $\{(G, H_1, H_2) : G \to (H_1, H_2)$ is Π_2^p-complete.
See `http://www.cs.umd.edu/~gasarch/COURSES/858/S20/notes/npramsey.pdf`

Complexity: NP-Completeness of Grid Extension

Grid Color Extension (GCE) is the set of tuples (n, m, c, χ) such that the following hold:

- $n, m, c \in \mathbb{N}$. χ is a partial c-coloring of $[n] \times [m]$ that is rectangle-free.
- χ can be extended to a rectangle-free coloring of $[n] \times [m]$.

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Thm (Apon, Gasarch, Lawler) *GCE* is NP-complete
`https://arxiv.org/pdf/1205.3813.pdf`

Complexity: Long Proofs Required

Def Resolution proofs are a proof system to show that a Boolean Formula is NOT satisfiable. It is of interest to find a class of non-satisfiable formulas ϕ_n that require (say) $(1.5)^n$ long Res Proofs.

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Def A graph is c-random if it does not contain a clique or ind set of size $c \log n$.

Def $\phi_{n,c}$ is a Boolean Formula that says **every** graph on n vertices is c-random. (This is false for c around $\frac{1}{2}$.)

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Def $\phi_{n,c}$ is a Boolean Formula that says **every** graph on n vertices is c-random. (This is false for c around $\frac{1}{2}$.)

Lauria, Pudlak, Rodl, Thapen proved:
Thm For appropriate c, any resolution proof for $\phi_{n,c}$ requires length $n^{\Omega(\log n)}$.
`https://arxiv.org/pdf/1303.3166.pdf`

PROS and CONS of Complexity of Ramsey

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Research What we **really** want is evidence that computing $R(k)$ is hard. These results do not really do that. Maybe you can!

Research Look at the above results for particular cases and see if easier.

Results Bill Does Not Care About But Should:VDW

Exposition by William Gasarch

May 18, 2020

Rado's Thm for Matrices

Rado's Thm Let $a_1, \ldots, a_k \in \mathbb{Z}$. TFAE

- Some subset of the a_i's sums to 0.
- For all c, for all $\mathrm{COL}\colon \mathbb{N} \to [c]$ there exists mono solution to

$$x_1 a_1 + \cdots + a_k x_k = 0.$$

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For a statement of the thm see the Wikipedia entry.

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See Wikipedia Entry for Statement.

This is someone else's slides on it. So I REALLY could have covered it!
`https://www.ti.inf.ethz.ch/ew/courses/extremal04/razen.pdf`

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https://www.ams.org/journals/tran/1971-159-00/S0002-9947-1971-0284352-8/S0002-9947-1971-0284352-8.pdf

Results Bill Does Not Care About But Should:Ramsey

Exposition by William Gasarch

May 18, 2020

Ramsey Over the Reals Fails: So what to do?

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- Ramsey Cardinals on Next Slide.

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Def κ is **inaccessible** if $\alpha < \kappa \implies 2^\alpha < \kappa$.

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Thm If κ is Ramsey then κ is inaccessible. (The converse is ind of ZFC but reasons to think its false.)

Results Bill May One Day Learn But Still too Hard for the Students

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May 18, 2020

Ramsey's Thm with control of the differences

Thm For all c, k and for all order types η there exists $N = N(c)$ such that for all $\mathrm{COL}\colon [N] \rightarrow [c]$ there exists a homog set $a_1 < \cdots < a_k$ such that

$$(a_2 - a_1, a_3 - a_2, \ldots, a_n - a_{n-1})$$

are all distinct and are in order type η.

- First proven by Noga Alon and Jan Pach using VDW, so bounds on $N(c)$ are large. Later Noga Alon, Alan Stacey, and Saharon Shelah got an iterated exp bound. None of this is written down anywhere.
- In 1995 Saharon Shelah got double exp bounds `https://arxiv.org/pdf/math/9502234.pdf`
- Shelah's paper is hard. I'm looking for easier proof of weaker results.

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Research Easier Proof.

Caveat There is a proof of Sz thm for Hales-Jewitt which is said to be elementary.
`https://arxiv.org/abs/0910.3926`

Green-Tao Thm

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Research Look for the AP's.