BILL, RECORD LECTURE!!!!

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Finite Ramsey Theorem For 3-Hypergraph

Exposition by William Gasarch

February 13, 2025

Notation

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We do an example of the first few steps of the construction.

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What to make of this? Discuss.

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We are given $\mathrm{COL}\colon \binom{[n]}{3} \to [2]$.

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For all $y, z \in H_1$, $\mathrm{COL}(1,y,z) = c_1$.

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If $y \in H_1$ we say that y **agrees**.
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Kill all those who disagree!

Construction of $x_1, H_1, c_1, x_2, H_2, c_2$

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We now have $x_1 = 1$.

H_1: for all $y, z \in H_1$, $\mathrm{COL}(x_1, y, z) = c_1$.

x_2 is the least element of H_1.

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Next Slide is General Case.

Construction of $x_{s+1}, H_{s+1}, c_{s+1}$

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H_{s+1} is the homog set from COL'. **Key** $|H_{s+1}| \geq \Omega(\log(|H_s|))$.

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Iterate this process $2k - 1$ times.

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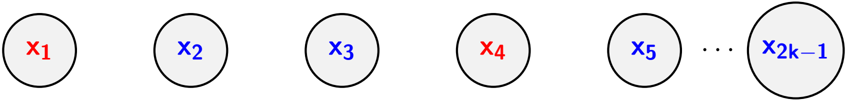

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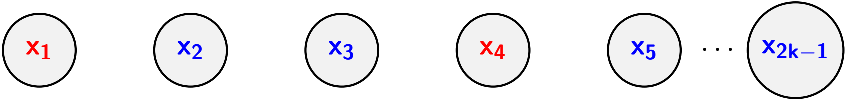

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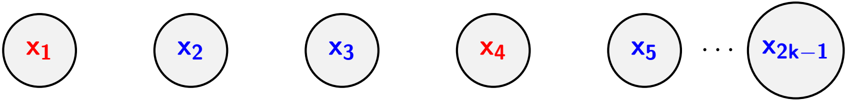

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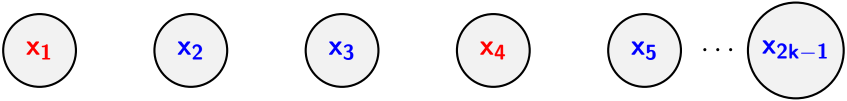

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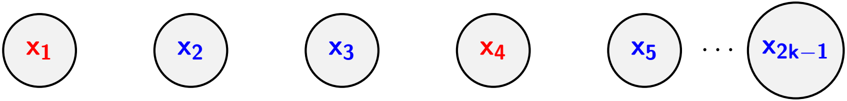

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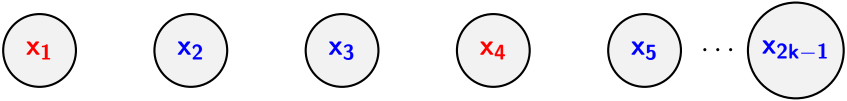

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What do you think our next step is?

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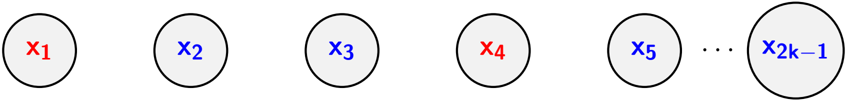

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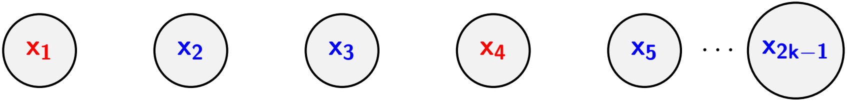

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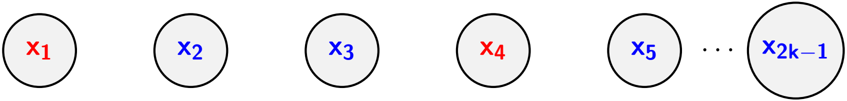

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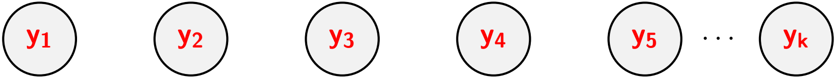

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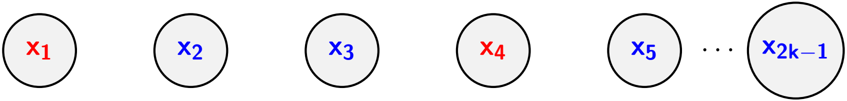

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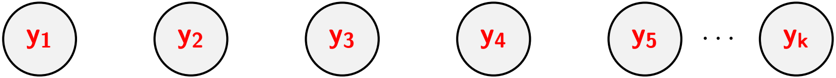

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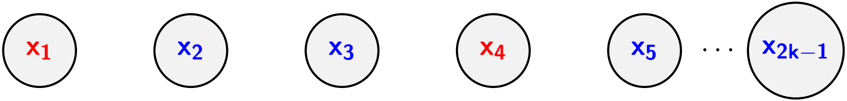

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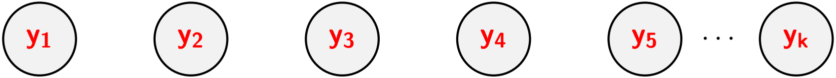

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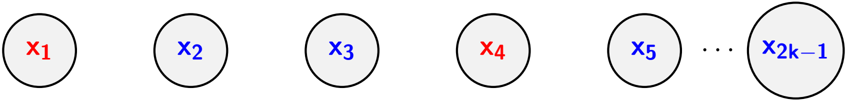

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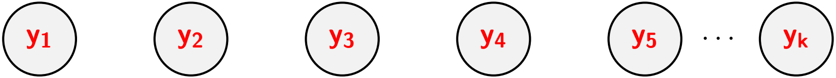

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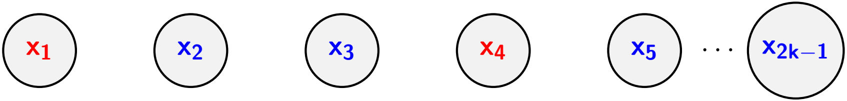

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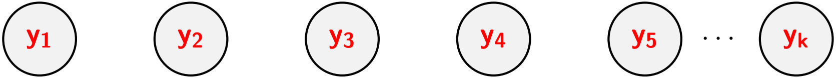

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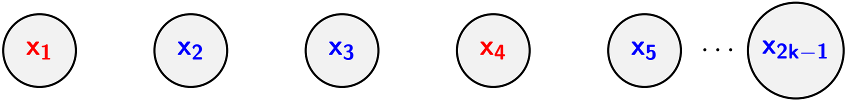

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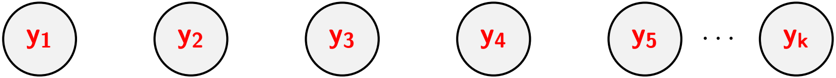

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Question Better Bounds?

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Beyond that the functions have no name.

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Thm $(\forall a)(\forall k)(\exists n)$ such that $(\forall \mathrm{COL}\colon \binom{[n]}{a} \rightarrow [2])$ there exists an homog set of size k. Our proof yields $n \le TOW_2(2k-1)$.

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Normally I would Vote now, but I need the terminology to state the bounds for a-hypergraph Ramsey Bounds.