BILL, RECORD LECTURE!!!!

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Infinite Ramsey Theorem For 3-Hypergraph

Exposition by William Gasarch

February 5, 2025

Notation

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We do an example of the first few steps of the construction.

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What to make of this?

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What to make of this? Discuss.

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We are given $\mathrm{COL}\colon \binom{\mathbb{N}}{3} \to [2]$.

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If $y \in H_1$ we say that y **agrees**.
If $y \notin H_1$ we say that y **disagrees**.

Kill all those who disagree!

Construction of $x_1, H_1, c_1, x_2, H_2, c_2$

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x_2 is the least element of H_1.

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H_2 is the homog set.

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Next Slide is General Case.

Construction of $x_{s+1}, H_{s+1}, c_{s+1}$

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H_{s+1} is the infinite homog set from COL'.

c_{s+1} is the color of H_{s+1}.

The Coloring of the Nodes

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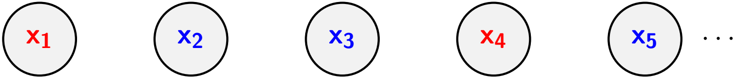

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x_1 x_2 x_3 x_4 x_5 $\cdots$

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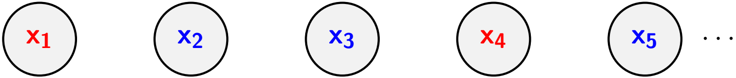

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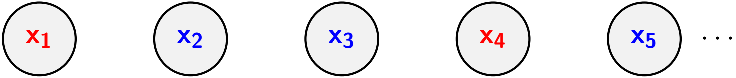

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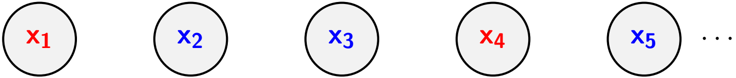

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What do you think our next step is?

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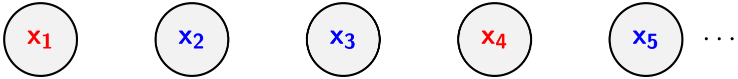

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$a \ge 4$: Might be a HW. Should be easy for you now.

Infinite to Finite

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We proved finite 2-ary Ramsey FROM infinite 2-ary Ramsey.

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That proof did not give any bounds on $R_2(k)$.

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That proof easily extends to $R_a(k)$.