The Blocks of 5 HS Math Comp Problem

Exposition by William Gasarch-U of MD

Prob 4 of UMCP HS Math Comp. 2025

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Thats where **you** come in!