Blocks of Five

Exposition by William Gasarch-U of MD

Part I:

A UMCP Math Competition Problem

And Its Generalization

Problem 4 From the UMCP Math Competition

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Find $m \in \mathbb{N}$ such that the following are true

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1) There exists an awesome collection of m blocks.

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Problem

Find $m \in \mathbb{N}$ such that the following are true
1) There exists an awesome collection of m blocks.
2) There does not exist an awesome collection of $m+1$ blocks.

Answer to Problem 4

We use the blocks

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Are we done?

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Are we done? **No!**

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Are we done? **No!** Need to show

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Are we done? **No!** Need to show there is no set of ≥ 507 blocks that works.

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Are we done? **No!** Need to show there is no set of ≥ 507 blocks that works. We first generalize the problem.

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If $1 \in A_3 \cap A_i$ then $\{1, n\} \subseteq A_1 \cap A_i$.

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$|T_2| \leq 16$.

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1) Since $|T_2| \leq 3$, the number of blocks that have n is ≤ 5.

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2) $m \le 2 + |T_1| + |T_2| \le 2 + 3 + 16 = 21$.

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Hence $m = 506$ is the answer.

The UMCP Problem: We Proved a Stronger Result

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What We Needed

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This leads to our general problem.

The Function $f(n)$

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Question For all $n \geq 5$ determine $f(n)$.

Part II:

Generalize the UMCP Solution

To Determine $f(n)$ for $n \geq 106$

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Side Project We will soon improve this bound to $n \leq 33$ using **complicated** though elementary techniques. Try to get a better-than-105 bound on n that uses **simple** elementary techniques.

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Part III:

Applying Equations To Our Problem

We Determine $f(n)$ for $33 \leq n \leq 105$

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Notation The phrase
Let $A_1, \ldots, A_m$ be an n-awesome* collection of blocks
means a collection of blocks where every $y \in [n]$ appears ≤ 5 times.

Needed Defs and Notation

Def Let $A_1, \ldots, A_m$ be an n-awesome* collection of blocks. An **incidence** is an ordered pair $((y, i) \in [n] \times [m]$ such $y \in A_i$.

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WRITE DOWN the Def and Notation!

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Lemma $\sum_{i\in[5]} n_i = n$. **WRITE THIS DOWN!**

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$n_1 = 8$ via $\{2, 4, 6, 8, 11, 12, 13, 14\}$. Each contributes 1 to the number of incidences, so total contribution is $1 \times n_1 = 8$.

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Example and Thm

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2) $(n_1, \ldots, n_5)$ is a solution in $\mathbb{Z}^5$ iff $\exists p, q \in \mathbb{Z}$ such that:

Summary of Lemmas Plus...

Lemma Let $m, n \in \mathbb{N}$. Assume $A_1, \ldots, A_m$ is an n-awesome* collection of blocks.

1) For $1 \le i \le 5$ n_i is the numb of $y \in [n]$ that appear i times.
$\sum_{i \in [5]} n_i = n$.
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2) $(n_1, \ldots, n_5)$ is a solution in $\mathbb{Z}^5$ iff $\exists p, q \in \mathbb{Z}$ such that:

$$n_1 = 0.5m^2 - 10.5m + 3n - p - 3q$$

$$n_2 = -m^2 + 16m - 3n + 3p + 8q$$

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$$n_4 = p$$

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(Proof by more algebra than you want to do.)

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Since $n \in \mathbb{N}$, $n \leq 33$.

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Case 2 There is no such y. Then $n \leq 33$, contrary to hypothesis.

Part IV:

Using Programs

We Determine $f(n)$ for $5 \leq n \leq 32$

GEN-NI PROGRAM. Generates all $(n_1, n_2, n_3, n_4, n_5)$

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If $(\forall i)[0 \le n_i \le 5]$ **then**

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If we eliminate **some** of them we may have insight into how to construct a 22-awesome* collection of 17 blocks.

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Blocktypes are 5-tuples from $\{1, 2, 3, 4, 5\}$ that add to $m + 4 = 21$.

$[5, 5, 5, 5, 1]$. **No!** Has 4 5's, but $n_5 = 3 < 4$.
$[5, 5, 5, 4, 2]$. **Fine!**
$[5, 5, 5, 3, 3]$. **No!** has 2 3's but $n_3 = 1 < 2$.

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Notation bt is a block type. If $1 \leq i \leq 5$ then bt^i is how many i's are in bt.

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We rephrase the argument from the last slide.

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Notation bt is a block type. If $1 \le i \le 5$ then bt^i is how many i's are in bt.

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Since $bt_1^5 = 4$ and $x_1 = 1$ there are at least 1×4 Mult-5 elts.

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This is **not** a contradiction.

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This is **not** a contradiction.

This does mean that bt_1 is not a valid blocktype.
General Theorem on next slide.

Elim-bt Theorem

Thm Assume there is an n-awesome* collection.

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Let bt be a blocktype.
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bt is called **invalid**

If $bt^i > n_i$ then bt has $> n_i$ mult-i elts which is a contradiction.

GEN-BT PROGRAM

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Sketch of Code

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Input $(n, m; n_1, n_2, n_3, n_4, n_5)$

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Input $(n, m; n_1, n_2, n_3, n_4, n_5)$

TABLE-BT $= \emptyset$

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Input $(n, m; n_1, n_2, n_3, n_4, n_5)$

$\text{TABLE-BT} = \emptyset$

For $b_1 = 5$ downto 1

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Sketch of Code

Input $(n, m; n_1, n_2, n_3, n_4, n_5)$

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For $b_1 = 5$ downto 1

    For $b_2 = b_1$ downto 1

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Sketch of Code

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Sketch of Code

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  For $b_2 = b_1$ downto 1

    For $b_3 = b_2$ downto 1

      For $b_4 = b_3$ downto 1

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For $b_1 = 5$ downto 1

  For $b_2 = b_1$ downto 1

    For $b_3 = b_2$ downto 1

      For $b_4 = b_3$ downto 1

        $b_5 = m + 4 - b_1 - b_2 - b_3 - b_4$

GEN-BT PROGRAM

Sketch of Code

Input $(n, m; n_1, n_2, n_3, n_4, n_5)$

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For $b_1 = 5$ downto 1

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    For $b_3 = b_2$ downto 1

      For $b_4 = b_3$ downto 1

        $b_5 = m + 4 - b_1 - b_2 - b_3 - b_4$

        if $1 \leq b_5 \leq 5$ then

GEN-BT PROGRAM

Sketch of Code

Input $(n, m; n_1, n_2, n_3, n_4, n_5)$

$\text{TABLE-BT} = \emptyset$

```
For b1 = 5 downto 1
    For b2 = b1 downto 1
        For b3 = b2 downto 1
            For b4 = b3 downto 1
                b5 = m + 4 - b1 - b2 - b3 - b4
                if 1 <= b5 <= 5 then
                    bt = [b1, b2, b3, b4, b5]
```

GEN-BT PROGRAM

Sketch of Code

Input $(n, m; n_1, n_2, n_3, n_4, n_5)$

TABLE-BT $= \emptyset$

```
For b_1 = 5 downto 1
    For b_2 = b_1 downto 1
       For b_3 = b_2 downto 1
          For b_4 = b_3 downto 1
             b_5 = m + 4 - b_1 - b_2 - b_3 - b_4
             if 1 ≤ b_5 ≤ 5 then
                bt = [b_1, b_2, b_3, b_4, b_5]
                If (∀1 ≤ i ≤ 5)[bt^i ≤ n_i] then
```

GEN-BT PROGRAM

Sketch of Code

```
Input (n, m; n_1, n_2, n_3, n_4, n_5)
TABLE-BT = ∅
For b_1 = 5 downto 1
    For b_2 = b_1 downto 1
        For b_3 = b_2 downto 1
            For b_4 = b_3 downto 1
                b_5 = m + 4 − b_1 − b_2 − b_3 − b_4
                if 1 ≤ b_5 ≤ 5 then
                    bt = [b_1, b_2, b_3, b_4, b_5]
                    If (∀1 ≤ i ≤ 5)[bt^i ≤ n_i] then
                        TABLE-BT = TABLE-BT ∪ {bt}
```

Note Output is a **set** of blocktypes.

GEN-BT PROGRAM

Sketch of Code

Input $(n, m; n_1, n_2, n_3, n_4, n_5)$

$\mathrm{TABLE\text{-}BT} = \emptyset$

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For b1 = 5 downto 1
   For b2 = b1 downto 1
      For b3 = b2 downto 1
         For b4 = b3 downto 1
            b5 = m + 4 - b1 - b2 - b3 - b4
            if 1 ≤ b5 ≤ 5 then
               bt = [b1, b2, b3, b4, b5]
               If (∀1 ≤ i ≤ 5)[bt^i ≤ n_i] then
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Note Output is a **set** of blocktypes.

Side project Do this program more efficiently.

Example of Elimination Via Equations

For $n = 17$ $m = 7$. $(n_1, n_2, n_3, n_4, n_5) = (6, 9, 2, 0, 0)$.

Example of Elimination Via Equations

For $n = 17$ $m = 7$. $(n_1, n_2, n_3, n_4, n_5) = (6, 9, 2, 0, 0)$.
$A_1 = \{1, 2, 3, 4, 5\}$ $A_2 = \{1, 6, 7, 8, 9\}$ $A_3 = \{1, 10, 11, 12, 13\}$
$A_4 = \{2, 6, 10, 14, 15\}$ $A_5 = \{2, 7, 11, 16, 17\}$ $A_6 = \{3, 8, 12, 14, 16\}$

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$r_1 = 3$ $r_2 = 3$ $r_3 = 2$ $r_4 = 1$ $r_5 = 1$ $r_6 = 2$
$r_7 = 2$ $r_8 = 2$ $r_9 = 1$ $r_{10} = 2$ $r_{11} = 2$ $r_{12} = 2$
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Block Types

A_1 has blocktype $[3, 3, 2, 1, 1]$
A_2, A_3,A_4,A_5 have blocktype $[3, 2, 2, 2, 1]$
A_6 has blocktype $[2, 2, 2, 2, 2]$

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$n_1 = 6$. 6 elts appear 1 time. 1 appears 1×6 times in blocktypes.

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General i appears $i \times n_i$ times in blocktypes.

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$n = 22, \ m = 17.$

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$n = 22$, $m = 17$. We consider $(n_1, n_2, n_3, n_4, n_5) = (0, 3, 7, 2, 10)$.

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Blocktypes are 5-tuples of numbers in $\{1, 2, 3, 4, 5\}$ that sum to 21:
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$[5, 5, 4, 4, 3]$
$[5, 4, 4, 4, 4]$. **No!**

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Example of Eliminate Via Equations

$n = 22$, $m = 17$. We consider $(n_1, n_2, n_3, n_4, n_5) = (0, 3, 7, 2, 10)$.

Each block type must sum to $m + 4 = 21$.

Blocktypes are 5-tuples of numbers in $\{1, 2, 3, 4, 5\}$ that sum to 21:
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There are **no** blocks of type $[5, 4, 4, 4]$ since there are only 2 elements of multiplicity 4.

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Continue this exciting story on the next slide.

Eliminating $(0, 3, 7, 2, 10)$

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Algebra shows these equations have no solution at all!

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Algebra shows these equations have no solution at all!

Only needed that it had no solution in $\mathbb{N}$.

GEN-EQ PROGRAM

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Sketch of Code

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Input $[[bt_1], \ldots, [bt_e]]$. A set of blocktypes.

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We call this set of equations **the usual equations**.

GEN-SOL PROGRAM

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Sketch of Code

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Input $\langle eq_1, \ldots, eq_6 \rangle$ A set of 6 linear equations.

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Input $\langle eq_1, \ldots, eq_6 \rangle$ A set of 6 linear equations.
Use a package to find out about solutions.

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If there is is a solution but not a solution over $\mathbb{N}$ then output $\mathrm{NONE}\text{-}\mathbb{N}$.

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If there are solutions over $\mathbb{N}$ then output all of them.

Eliminating a Solution

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$n = 22,\ m = 17,\ (n_1, n_2, n_3, n_4, n_5) = (1, 1, 7, 4, 9).$

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Blocktypes:

x_1 $[5, 5, 5, 5, 1]$

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The equations:

Mult 1: $x_1 = 1 \times n_1 = 1$
Mult 2: $x_2 = 2 \times n_2 = 2$
Mult 3: $2x_3 + x_4 = 3 \times n_3 = 21$
Mult 4: $x_2 + 2x_4 + 4x_5 = 4 \times n_4 = 16$
Mult 5: $4x_1 + 3x_2 + 3x_3 + 2x_4 + x_5 = 5 \times n_5 = 45$
Blocks: $x_1 + x_2 + x_3 + x_4 + x_5 = 17$.

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Blocks: $x_1 + x_2 + x_3 + x_4 + x_5 = 17.$

There are 4 solutions: $\langle x_1, x_2, x_3, x_4, x_5 \rangle =$
$\langle 1, 2, 7, 7, 0 \rangle$, $\langle 1, 2, 8, 5, 1 \rangle$, $\langle 1, 2, 9, 3, 2 \rangle$, $\langle 1, 2, 10, 1, 3 \rangle$

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We show $\langle 1, 2, 9, 3, 2 \rangle$ does not work on next slide.

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Solution $\langle x_1, x_2, x_3, x_4, x_5 \rangle = \langle 1, 2, 9, 3, 2 \rangle$.
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These pairs are distinct since all intersections are of size 1.

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Numb of pairs of elts is $\geq 6 + 6 + 27 + 3 = 42$.

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But $\binom{n_5}{2} = \binom{9}{2} = 36 < 42$.

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These pairs are distinct since all intersections are of size 1.

Numb of pairs of elts is $\geq 6 + 6 + 27 + 3 = 42$.
But $\binom{n_5}{2} = \binom{9}{2} = 36 < 42$. Contradiction.

Example of Elim using Equations

Recall bt is a block type. If $1 \leq i \leq 5$ then bt^i is how many i's are in bt.

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Recall bt is a block type. If $1 \le i \le 5$ then bt^i is how many i's are in bt.

Example $m = 22$, $m = 17$, $(n_1, n_2, n_3, n_4, n_5) = (1, 1, 7, 4, 9)$.
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Example of Elim using Equations

Recall bt is a block type. If $1 \le i \le 5$ then bt^i is how many i's are in bt.

Example $m = 22$, $m = 17$, $(n_1, n_2, n_3, n_4, n_5) = (1, 1, 7, 4, 9)$.

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The number of pairs of Mult-5 elts is at least

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General Theorem on next slide.

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3) $\forall$ solution $\langle x_1, \ldots, x_e \rangle$ to the usual equations, $\exists 1 \le i \le 5$:

$$x_1 \binom{bt_1^i}{2} + \cdots + x_e \binom{bt_e^i}{2} > \binom{n_i}{2}.$$

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SOL-BAD PROGRAM

Input $(n, m; n_1, n_2, n_3, n_4, n_5)$,$[[bt_1], \ldots, [bt_e]]$, $\langle x_1, \ldots, x_e \rangle$

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        SOLGOOD=FALSE
Output SOLGOOD (this will be TRUE or FALSE)

*m*WORKS? PROGRAM

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    $\mathrm{TABLE\text{-}EQ} = \mathrm{GEN\text{-}EQ}[[bt_1], \ldots, [bt_e]]$

    For all $\langle eq_1, \ldots, eq_6 \rangle \in \mathrm{TABLE\text{-}EQ}$

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   $\mathrm{TABLE\text{-}SOL} = \mathrm{GEN\text{-}SOL}(\langle eq_1, \ldots, eq_6 \rangle)$
   For all $\langle x_1, \ldots, x_e \rangle \in \mathrm{TABLE\text{-}SOL}$

$\mathrm{SOL\text{-}GOOD} =$
$\mathrm{SOL\text{-}BAD}(n, m; n_1, n_2, n_3, n_4, n_5), [[bt_1], \ldots, [bt_e]], \langle x_1, \ldots, x_e \rangle$
  If $\mathrm{SOL\text{-}GOOD} \neq \mathrm{FALSE}$ then output

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  For all $[[bt_1], \ldots, [bt_e]]$ in $\mathrm{TABLE\text{-}BT}$

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  For all $[[bt_1], \dots, [bt_e]]$ in TABLE-BT
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    For all $\langle eq_1, \dots, eq_6 \rangle \in$ TABLE-EQ
      TABLE-SOL = GEN-SOL$(\langle eq_1, \dots, eq_6 \rangle)$
      For all $\langle x_1, \dots, x_e \rangle \in$ TABLE-SOL

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    If SOL-GOOD $\neq$ FALSE then output
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      $\langle x_1, \dots, x_e \rangle$
      else output FALSE

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    $x = m\text{WORKS?}(n, m)$

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For $m = 1$ to 21
    $x = m\mathrm{WORKS?}(n, m)$
If x is not FALSE then output all of the information x had.

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We are not done yet. See Next Slide.

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Which am I rooting for? I will be happy with either:
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b) We find some math of interest and THEN we are DONE.
I will be unhappy if we can't actually solve the problem.