BILL, RECORD LECTURE!!!!

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Getting Many Mono K_4's: Two Approaches

William Gasarch

Lets Party Like Its 2019

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Thm For all 2-col of the edges of K_{18} there is a mono K_4.

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{36} there are 2 mono K_4's

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1. As posed the answer is $n = 18$. Piwakoswki and Radziszowski `https://www.cs.rit.edu/~spr/PUBL/paper40.pdf` showed that for every 2-col of K_{18} there are 9 mono K_4's.

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2. I will present a two math-interesting proof of the following: **Thm** For all 2-cols of K_{19} there are TWO mono K_4's.
3. In both cases I will extend to getting many copies of K_4 two different ways.

The Garrett Peters Approach

William Gasarch

Proof of K_{19} Two K_4 Theorem

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Remove 19 to get K_{18}.

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DONE!

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IS) Let $\mathrm{COL}\colon \binom{[f(m)]}{2} \rightarrow [2]$. We assume $f(m) \geq 18$.

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SO $f(m) = m + 17$.

The Standard Approach

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Since $|A_i| = R(4)$, each A_i has a mono K_4.

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So we get $\binom{19}{18}$ mono K_4's. So we are almost there.

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For the real proof, see next slide.

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(There are just 19 of these, $A_i = \{1, \ldots, 19\} - \{i\}$.)

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There are $\binom{19}{18} - \binom{15}{14} = 19 - 15 = 4$ left. Call then B_1, B_2, B_3, B_4.

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4) The mono K_4 from A_1, and the mono K_4 from B are different.

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Those are our 2 mono K_4's.

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1) Find 1st mono K_4 in A_1. Say its $\{x_1, x_2, x_3, x_4\}$.

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$\binom{n-4}{18-4} = \binom{n-4}{14}$ of these. There are $\binom{n}{18} - \binom{n-4}{14}$ left.

3) Find 2nd mono K_4 in one of the sets left. Say its $\{y_1, y_2, y_3, y_4\}$.

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$\binom{n-4}{18-4} = \binom{n-4}{14}$ of these. There are $\binom{n}{18} - 2\binom{n-4}{14}$ left.

Want n such that $\forall$ 2-col $\exists 3$ Mono K_4's

$\forall$ 2-col of K_n $\exists$ 3 mono K_4's.

List out all subsets of $V = \{1, \ldots, n\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{n}{18}}.$$

1) Find 1st mono K_4 in A_1. Say its $\{x_1, x_2, x_3, x_4\}$.

2) REMOVE all A_i's that have all of $\{x_1, x_2, x_3, x_4\}$.
$\binom{n-4}{18-4} = \binom{n-4}{14}$ of these. There are $\binom{n}{18} - \binom{n-4}{14}$ left.

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$\binom{n-4}{18-4} = \binom{n-4}{14}$ of these. There are $\binom{n}{18} - 2\binom{n-4}{14}$ left.

5) Find 3rd mono K_4 in one of the sets left. DONE.

Want 3 Mono K_4's (cont)

Need

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$$\binom{n}{18} - 2\binom{n-4}{14} \geq 1$$

Want 3 Mono K_4's (cont)

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Want 3 Mono K_4's (cont)

$$n(n-1)(n-2)(n-3) > 2 \times 18 \times 17 \times 16 \times 15 = 146889.$$

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n	$n(n-1)(n-2)(n-3)$
19	93024
20	116280
21	143640
22	175560

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Garrett Peters got:
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Since $\binom{n}{18} - (m-1)\binom{n-4}{14} \geq 1$ this process can go for $\geq m$ iterations and produce $\geq m$ mono K_4's.

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Want m Mono K_4's (cont)

We just proved the following.

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Thm Let $g(m)$ be the least n such that

$$n \times (n-1) \times (n-2) \times (n-3) > 73440(m-1)$$

Then $\forall$ 2-col of $K_{g(m)}$ $\exists$ m mono K_4's.

Compare Garrett Peters (GP) to Standard (ST)

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Extra Credit Improve the standard approach so that the crossover point is lower.

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3) There are $\frac{n(n-1)(n-2)(n-3)}{73440} + 1$ mono K_4's.

4) There are $\frac{n^4}{73440} - \frac{n^3}{12240} + \Omega(n^2)$ mono K_4's.

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There are $\frac{1}{3060}\binom{n}{4}$ mono K_4's.

Generalize

Left to the reader

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1. Generalize to mono K_m.

Generalize

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1. Generalize to mono K_m.
2. Generalize to c colors.

Generalize

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1. Generalize to mono K_m.
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3. Generalize to c colors and mono K_m.