BILL, RECORD LECTURE!!!!

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A Variant on $R(3) = 6$

Exposition by William Gasarch

April 1, 2025

Credit Where Credit Is Due

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The questions raised in these slides are due to Paul Erdös.

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Shen Lin.

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Vote: YES or NO or UNKNOWN TO SCIENCE.

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Answer on next slide.

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Closing that gap is open.

G Such That $\mathrm{RAM}(G)$, G Has No K_5 Subgraph, G Has 18 Vertices

Detour:
Vertex Ramsey Theory

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We could also look at coloring **vertices**.

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Not so fast! What if we start a graph other than K_n?

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Similar to our study of G where 2-coloring edges yields 3-homog set.

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We will use a result in Vertex-Ramsey to help Graph Ramsey.

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That will be a HW. Irving's paper may help:
`http://www.cs.umd.edu/~gasarch/TOPICS/grt/irving.pdf`

Back to
Graph Ramsey Theory

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If the K_5 does have v_0 then remove v_0 and you have that K_4 is a subgraph of H, contradiction.

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See next slide for pictures and grand finale!

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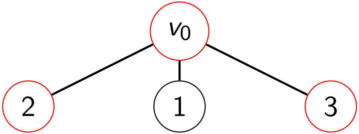

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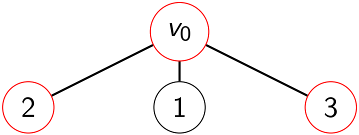

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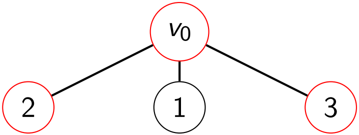

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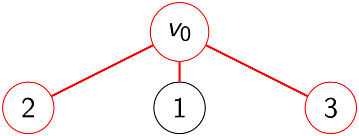

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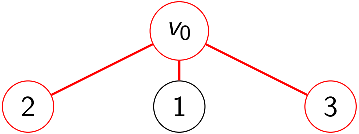

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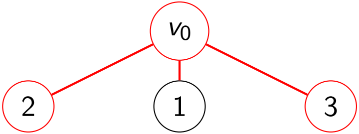

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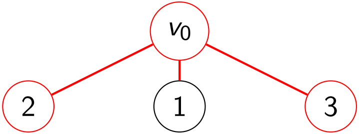

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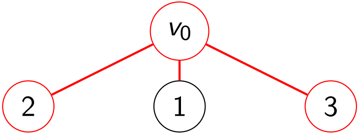

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Done!

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Upshot We will skip this; however, you can read my slides if you are curious.

Recall General Theorem

Thm Let $G = (V, E)$. If V can be partitioned into 5 ind. sets then $\exists\ \mathrm{COL}\colon E \rightarrow [2]$ with no mono $\triangle$.

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$V = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

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a) V can be partitioned into 5 ind. sets.
b) (Using Theorem) $\exists$ $\mathrm{COL}\colon E \rightarrow [2]$ with no mono $\triangle$s.

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So $\{1, 2, 3\}$ is a K_3.

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Finishing Up The Proof

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We are not going to go through the cases.

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Why Not

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We are not going to go through the cases.

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We Are Busy People!