BILL, RECORD LECTURE!!!!

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May 13, 2025

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- 7) Please Fill Out the Teaching Evals in All of your Courses

Topics Not Covered in Grad Ramsey 2025

Exposition by William Gasarch

May 13, 2025

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- Some combination of the above.

Could Have Covered: VDW

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May 13, 2025

Rado's Thm Let $a_1, \ldots, a_k \in \mathbb{Z}$. TFAE

- \triangleright Some subset of the a_i 's sums to 0.
- ▶ For all c, for all $COL: \mathbb{N} \to [c]$ there exists mono solution to

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Folkman's Thm For all k, c there exists N = N(k, c) such that for all COL: $[N] \rightarrow [c]$ there exists x_1, \ldots, x_k such that ALL non-empty sums of the x_i 's are the same color.

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Great thm, nice proof. Might cover it in the future.



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Find c such that

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- ► Caution: Some of this may be known.

Hilbert's Cube Lemma For all k, c there exists H = H(k, c) such that for all COL: $[H] \rightarrow [c]$ there exists x_0, x_1, \dots, x_k such that

$$\{x_0 + \sum_{i=1}^k b_i x_i : b_i \in \{0, 1\}\}$$

is monochromatic.

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H Irreducibility Thm (2 var case). If $p(x,y) \in \mathbb{Q}[x,y]$ is irred then there exists $a \in \mathbb{Z}$ such that $p(x,a) \in \mathbb{Q}[x]$ is irred.

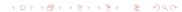
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- I've taught before and could teach again.

Rado's Theorem over the Reals

Vote

For all COL: $\mathbb{R} \to \mathbb{N}$ there exists w, x, y, z all the same color:

$$w + x = y + z$$

- ► TRUE
- ► FALSE
- ► OTHER

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Proven by Erdos. Write up by Fenner and Gasarch is here: http://www.cs.umd.edu/~gasarch/BLOGPAPERS/radozfc.pdf

Could have Covered: Ramsey

Exposition by William Gasarch

May 13, 2025

 $R(C_k)$ is least n such that for all 2-coloring of $\binom{[n]}{2}$ there exists monochromatic k-cycle.

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Sample Thm

$$R(C_k) = \begin{cases} 6 & \text{if } k = 3 \text{ or } k = 4\\ 2k - 1 & \text{if } k \ge 5 \text{ and } k \equiv 1 \pmod{2} \\ \frac{3k}{2} - 1 & \text{if } k \ge 6 \text{ and } k \equiv 0 \pmod{2} \end{cases}$$
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- ► For every result of this type see https://www.combinatorics.org/files/Surveys/ds1/ ds1v15-2017.pdf

Research Projects

- Actually FIND the colorings.
- ► Simplify or unify the proofs
- **Ramsey Games** Example: Parameter k, n. Players RED and BLUE alternate coloring the edges of K_n . RED goes first. The first player to get a C_k in their color wins.
 - 1. For which *n* does RED have a winning strategy?
 - 2. Design an ML to play this well (my REU project)
 - EVERY thm in Ramsey Thm (and the VDW part) can be made into a game and lead to research projects.

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Research Use their technique on other Ramsey problems.

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Large Can Ramsey:

The following is well known; however, I may be the first person to write down the proof.

http://www.cs.umd.edu/~gasarch/COURSES/858/S20/notes/canlarge.pdf

Thm For all k there exists n=n(k) such that for all $COL: \binom{\{k,\dots,n\}}{2} \to [\omega]$ there is a large set that is either homog, min-homog, max-homog, rainbow.

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a—ary Can Ramsey: I don't want to state it since its complicated. Similar to the proof on graphs, but messier.

Optimal results due to Shelah:

https://arxiv.org/abs/math/9509229 A hard read.



More Euclidean Ramsey Theory

Exposition by William Gasarch

May 13, 2025

Sample Thm Let T be a triangle with a 30, 90, or 150 degree angle. For every 2-coloring of \mathbb{R}^2 there exists three points that form triangle T (note- actually form T, not just similar to T) that are monochromatic.

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- ► For more: https://www.csun.edu/~ctoth/Handbook/chap11.pdf

Results Bill Likes But Would be Hard to Teach: VDW

Exposition by William Gasarch

May 13, 2025

Def *L* is a language. Game:

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- ▶ Alice is Poly time and she has x, |x| = n.
- Bob is all powerful and he has nothing.
- ▶ They cooperate to determine if $x \in L$. Alice could just send Bob x. That takes n bits.

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App of 3-Free Sets to Complexity Theory Cont

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- ► Too much prerequisite knowledge.

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- ▶ Research Come up with an elementary proof.

Results Bill Likes But Would be Hard to Teach:Ramsey

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Complexity: Π_2^p Completeness of Arrow

Def $G \to (H_1, H_2)$ means that for every 2-coloring of the edges of G there is either a **RED** H_1 or a **BLUE** H_2 .

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Marcus Schaefer proved the following.

Thm $\{(G, H_1, H_2) : G \rightarrow (H_1, H_2) \text{ is } \Pi_2^p\text{-complete.} \}$

See http://www.cs.umd.edu/~gasarch/COURSES/858/S20/notes/npramsey.pdf

Complexity: NP-Completeness of Grid Extension

Grid Color Extension (GCE) is the set of tuples (n, m, c, χ) such that the following hold:

- ▶ $n, m, c \in \mathbb{N}$. χ is a partial c-coloring of $[n] \times [m]$ that is rectangle-free.
- \triangleright χ can be extended to a rectangle-free coloring of $[n] \times [m]$.

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Thm (Apon, Gasarch, Lawler) *GCE* is NP-complete https://arxiv.org/pdf/1205.3813.pdf

Complexity: NP-Completeness of Grid Extension

Grid Color Extension (GCE) is the set of tuples (n, m, c, χ) such that the following hold:

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Soren said this was bullshit man!

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Lauria, Pudlak, Rodl, Thapen proved:

Thm For appropriate c, any resolution proof for $\phi_{n,c}$ requires length $n^{\Omega(\log n)}$.

https://arxiv.org/pdf/1303.3166.pdf

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Research What we really want is evidence that computing R(k) is hard. These results do not really do that. Maybe you can!
Research Look at the above results for particular cases and see if easier.

Results Bill Does Not Care About But Should:VDW

Exposition by William Gasarch

May 13, 2025

Rado's Thm Let $a_1, \ldots, a_k \in \mathbb{Z}$. TFAE

- ► Some subset of the a_i's sums to 0.
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For a statement of the thm see the Wikipedia entry.

A Very General Thm from which we can derive cleanly VDW and Gallai-Witt.

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This is someone else's slides on it. So I REALLY could have covered it!

https:

//www.ti.inf.ethz.ch/ew/courses/extremal04/razen.pdf

Ramsey's thm for n-parameter sets

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https://www.ams.org/journals/tran/1971-159-00/
S0002-9947-1971-0284352-8/S0002-9947-1971-0284352-8.
pdf
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Results Bill Does Not Care About But Should:Ramsey

Exposition by William Gasarch

May 13, 2025

Thm (AC) There is a coloring of $\binom{\mathbb{R}}{2}$ with no homog set of size \mathbb{R} . So what to do?

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- Ramsey Cardinals on Next Slide.

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Def κ is **inaccessible** if $\alpha < \kappa \implies 2^{\alpha} < \kappa$.

Ramsey Cardinals

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Thm If κ is Ramsey then κ is inaccessible. (The converse is ind of ZFC but reasons to think its false.)

Results Bill May One Day Learn But Still too Hard for the Students

Exposition by William Gasarch

May 13, 2025

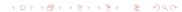
Ramsey's Thm with control of the differences

Thm For all c, k and for all order types η there exists N = N(c) such that for all COL: $[N] \to [c]$ there exists a homog set $a_1 < \cdots < a_k$ such that

$$(a_2-a_1,a_3-a_2,\ldots,a_n-a_{n-1})$$

are all distinct and are in order type η .

- ▶ First proven by Noga Alon and Jan Pach using VDW, so bounds on N(c) are large. Later Noga Alon, Alan Stacey, and Saharon Shelah got an iterated exp bound. None of this is written down anywhere.
- ► In 1995 Saharon Shelah got double exp bounds https://arxiv.org/pdf/math/9502234.pdf
- Shelah's paper is hard. I'm looking for easier proof of weaker results.



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Research Easier Proof.

Caveat There is a proof of Sz thm for Hales-Jewitt which is said to be elementary.

https://arxiv.org/abs/0910.3926

Thm For all k the set of primes has a k-AP.

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Research Look for the AP's.