BILL, RECORD LECTURE!!!!

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Please Fill Out All of Your Courses Teaching Evals

May 13, 2025

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7) **Please Fill Out the Teaching Evals in All of your Courses**

Topics Not Covered in Grad Ramsey 2025

Exposition by William Gasarch

May 13, 2025

We Didn't Cover X Because...

What topics in Ramsey theory didn't we cover?

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- Some combination of the above.

Could Have Covered: VDW

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Folkman's Thm

Rado's Thm Let $a_1, \ldots, a_k \in \mathbb{Z}$. TFAE

- Some subset of the a_i's sums to 0.
- For all c, for all $\mathrm{COL}\colon \mathbb{N} \to [c]$ there exists mono solution to

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Great thm, nice proof. Might cover it in the future.

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- Canonical Version of Rado or Folkman's Thm.
- Caution: Some of this may be known.

Hilbert's Cube Lemma

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$$\{x_0 + \sum_{i=1}^{k} b_i x_i : b_i \in \{0, 1\}\}$$

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- I've taught before and could teach again.

Rado's Theorem over the Reals

Vote

For all $\mathrm{COL}\colon \mathbb{R} \to \mathbb{N}$ there exists w, x, y, z all the same color:

$$w + x = y + z$$

- TRUE
- FALSE
- OTHER

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Proven by Erdos. Write up by Fenner and Gasarch is here:
`http://www.cs.umd.edu/~gasarch/BLOGPAPERS/radozfc.pdf`

Could have Covered: Ramsey

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- For every result of this type see `https://www.combinatorics.org/files/Surveys/ds1/ds1v15-2017.pdf`

Research Projects

- Actually FIND the colorings.
- Simplify or unify the proofs
- **Ramsey Games** Example: Parameter k, n. Players RED and BLUE alternate coloring the edges of K_n. RED goes first. The first player to get a C_k in their color wins.
  1. For which n does RED have a winning strategy?
  2. Design an ML to play this well (my REU project)
  3. EVERY thm in Ramsey Thm (and the VDW part) can be made into a game and lead to research projects.

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Research Use their technique on other Ramsey problems.

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`http://www.cs.umd.edu/~gasarch/COURSES/858/S20/notes/canlarge.pdf`

Thm For all k there exists $n = n(k)$ such that for all $\mathrm{COL}\colon \binom{\{k,\ldots,n\}}{2} \to [\omega]$ there is a large set that is either homog, min-homog, max-homog, rainbow.

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$a-$ary Can Ramsey: I don't want to state it since its complicated. Similar to the proof on graphs, but messier.
Optimal results due to Shelah:
https://arxiv.org/abs/math/9509229 A hard read.

More Euclidean Ramsey Theory

Exposition by William Gasarch

May 13, 2025

Euclidean Ramsey Theory

Sample Thm Let T be a triangle with a 30, 90, or 150 degree angle. For every 2-coloring of $\mathbb{R}^2$ there exists three points that form triangle T (note- actually form T, not just similar to T) that are monochromatic.

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- For more: `https://www.csun.edu/~ctoth/Handbook/chap11.pdf`

Results Bill Likes But Would be Hard to Teach:VDW

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App of 3-Free Sets to Complexity Theory

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Def L is a language. Game:

- Alice is Poly time and she has x, $|x| = n$.
- Bob is all powerful and he has nothing.
- They cooperate to determine if $x \in L$. Alice could just send Bob x. That takes n bits.

App of 3-Free Sets to Complexity Theory Cont

Let L be the set of all 3-colorable graphs (or any NPC graph problem). Note size is $O(n^2)$. Is there a protocol for Alice and Bob in $O(n^{2-\epsilon})$ bits for some $\epsilon > 0$?

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- **Research** Come up with an elementary proof.

Results Bill Likes But Would be Hard to Teach:Ramsey

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Marcus Schaefer proved the following.
Thm $\{(G, H_1, H_2) : G \rightarrow (H_1, H_2)$ is Π_2^p-complete.
See `http://www.cs.umd.edu/~gasarch/COURSES/858/S20/notes/npramsey.pdf`

Complexity: NP-Completeness of Grid Extension

Grid Color Extension (GCE) is the set of tuples (n, m, c, χ) such that the following hold:

- $n, m, c \in \mathbb{N}$. χ is a partial c-coloring of $[n] \times [m]$ that is rectangle-free.
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Soren said this was bullshit man!

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Def Resolution proofs are a proof system to show that a Boolean Formula is NOT satisfiable. It is of interest to find a class of non-satisfiable formulas ϕ_n that require (say) $(1.5)^n$ long Res Proofs.

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Lauria, Pudlak, Rodl, Thapen proved:
Thm For appropriate c, any resolution proof for $\phi_{n,c}$ requires length $n^{\Omega(\log n)}$.
https://arxiv.org/pdf/1303.3166.pdf

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Research What we **really** want is evidence that computing $R(k)$ is hard. These results do not really do that. Maybe you can!
Research Look at the above results for particular cases and see if easier.

Results Bill Does Not Care About But Should:VDW

Exposition by William Gasarch

May 13, 2025

Rado's Thm for Matrices

Rado's Thm Let $a_1, \ldots, a_k \in \mathbb{Z}$. TFAE

- Some subset of the a_i's sums to 0.
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For a statement of the thm see the Wikipedia entry.

Hales-Jewitt Thm

A Very General Thm from which we can derive cleanly VDW and Gallai-Witt.

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This is someone else's slides on it. So I REALLY could have covered it!
`https://www.ti.inf.ethz.ch/ew/courses/extremal04/razen.pdf`

Ramsey's thm for n-parameter sets

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https://www.ams.org/journals/tran/1971-159-00/S0002-9947-1971-0284352-8/S0002-9947-1971-0284352-8.pdf

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Exposition by William Gasarch

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- Prove what you can: If κ is a cardinal then for all $\mathrm{COL}\colon \binom{2^{\kappa}+}{2}$ there is a homog set of size κ.
- Ramsey Cardinals on Next Slide.

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What About... Can ZFC prove that such a κ does not exist? Unknown.
Def κ is **inaccessible** if $\alpha < \kappa \implies 2^\alpha < \kappa$.

Ramsey Cardinals

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True $\aleph_0$ is Ramsey.

Question Does there exist a Ramsey cardinal $\kappa > \aleph_0$?

Vote: YES, NO, or OTHER.

Thm If κ is Ramsey then κ is inaccessible. (The converse is ind of ZFC but reasons to think its false.)

Results Bill May One Day Learn But Still too Hard for the Students

Exposition by William Gasarch

May 13, 2025

Ramsey's Thm with control of the differences

Thm For all c, k and for all order types η there exists $N = N(c)$ such that for all $\mathrm{COL}\colon [N] \rightarrow [c]$ there exists a homog set $a_1 < \cdots < a_k$ such that

$$(a_2 - a_1, a_3 - a_2, \ldots, a_n - a_{n-1})$$

are all distinct and are in order type η.

- First proven by Noga Alon and Jan Pach using VDW, so bounds on $N(c)$ are large. Later Noga Alon, Alan Stacey, and Saharon Shelah got an iterated exp bound. None of this is written down anywhere.
- In 1995 Saharon Shelah got double exp bounds `https://arxiv.org/pdf/math/9502234.pdf`
- Shelah's paper is hard. I'm looking for easier proof of weaker results.

Szemeredi, Furstenberg, Gowers

Szemeredi, Furstenberg, Gowers have given different proofs of:
Sz Thm If A has upper pos density then, for all k, A contains a k-AP.

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Research Easier Proof.

Caveat There is a proof of Sz thm for Hales-Jewitt which is said to be elementary.
https://arxiv.org/abs/0910.3926

Green-Tao Thm

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Research Easier proof, perhaps of subcases.

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Research Look for the AP's.