Poly VDW Theorem Cheat Sheet II
A Very Useful Notation

Theorem 0.1. Poly VDW Theorem *Let $p_1, \ldots, p_k \in \mathsf{Z}[x]$ be such that $p_i(0) = 0$. Let $c \in \mathsf{N}$. Then $\exists\ W = W(p_1, \ldots, p_k; c)$ such that for all* COL: $[W] \to [c]$ $\exists a, d$, *such that*

$$a, a + p_1(d), \ldots, a + p_k(d) \textit{ same col.}$$

Notation 0.2. PVDW$(p_1(x), \ldots, p_k(x))$ means
$\forall c \in \mathsf{N}$, $\exists\ W = W(p_1, \ldots, p_k; c)$ such that
$\forall$ COL: $[W] \to [c]$ $\exists a, d$ such that

$$a, a + p_1(d), \ldots, a + p_k(d) \text{ same col.}$$

Notation 0.3. Let P be a finite subset of $\mathsf{Z}[x]$ such that $(\forall p \in P)[p(0) = 0]$.
Assume the max degree of a poly is d.
For $1 \le i \le d$ let n_i be the number of lead coefficients of polys in P of degree i.
The *index* of P is $(n_d, n_{d-1}, \ldots, n_1)$.

1. $\{x^4, 2x^4 + x^3, x^2, 2x^2, 100x^2, x, 100000x)$ has index $(2, 0, 3, 2)$.
2. $\{x^3, x^3 + \Box_1 x^2 + \Box_2 x, x^2 + x, 3x, 4x, 10x\colon -10^{100} \le \Box_1, \Box_2 \le 10^{100}\}$ has index $(1, 1, 3)$.

Notation 0.4.

1. We put an ordering on indices of the same length: $(m_d, \ldots, m_1) \le (n_d, \ldots, n_1)$ if $(\forall i)[m_i \le n_i]$.
2. We now allow ω as an entry in an index

Notation 0.5. For $n_d, \ldots, n_1 \in \mathsf{N} \cup \{\omega\}$ PVDW$(n_d, \ldots, n_1)$ means
$(\forall P \subseteq \mathsf{Z}[x])$, P of index $\le (n_d, \ldots, n_1)$, PVDW(P) is true.

1. VDW's Theorem is PVDW(ω).
2. The theorem about $a, a + d^2$ is PVDW$(1, 0)$.