

Take Home Part of 752 Midterm for Spring 2026
Due Before Class March 31. Dead Cat April 2

1. (0 points) What is your name

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2. (25 points)

For this problem you can use the following approximations to the truth.

Let $R_a(k)$ be the a -ary Ramsey numbers.

- $R_2(k) \leq 2^{2^k}$
- For all COL: $\binom{[n]}{2} \rightarrow [2]$ there is a homog set of size $\geq \log n$.
- $R_3(k) \leq 2^{2^{4k}}$.
- For all COL: $\binom{[n]}{3} \rightarrow [2]$ there is a homog set of size $\geq \log \log n$.
- We denote the inverse of $\log^{(i)}(m)$ by $TOW_i(k)$.

Recall that statement of the 4-ary Ramsey Theorem.

Thm ($\forall k$), there exists n such that, for all COL: $\binom{[n]}{4} \rightarrow [2]$ there exists a homog set of size k .

We now (finally) state the problem.

There are three ways to prove the finite 4-ary Ramsey Theorem.

We state the three ways. For each way, prove the finite 4-ary Ramsey Theorem using that way, and give the bounds on n as a function of k that that way yields.

In your three proofs you should do the following:

- The first 3 steps of the construction.
- Stage s of the construction which yields a set H_s .
- A lower bound on $|H_s|$.
- What n needs be as a function of k .

Here are the 3 ways:

- (a) Apply 3-ary finite Ramsey many times, then 1-ary Ramsey once.
- (b) Apply 2-ary finite Ramsey many times, then 2-ary Ramsey once.
- (c) Apply 1-ary finite Ramsey many times, then 3-ary Ramsey once.