

Homework 08, Morally Due 12:30PM, Tue Apr 7 2026

1. (0 points) What is your name?

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2. (30 points) Let $\text{COL}: \binom{[n]}{2} \rightarrow [\omega]$. Assume that, for all colors c , for all $x \in [n]$, $\deg_c(x) \leq 17$. Show that there is a rainbow set of size $\Omega(n^{1/3})$.

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3. (40 points) **Recall:** we showed the following:

For all $f: Z \times Z \rightarrow Z$ there exists infinite $D \subseteq Z$ such that f restricted to $D \times D$ is NOT onto.

Recall that we used the 1-ary Ramsey Theorem ONCE and the 2-ary Ramsey Theorem TWICE. We could that as THREE uses of Ramsey's Theorem.

And NOW for our problems

(a) Let $f: Z \times Z \times Z \rightarrow Z$.

Show there exists a $D \subseteq Z$ such that

f restricted to $D \times D \times D$ is not onto.

How many times does your proof use the Ramsey's theorem?

(b) Let $a \in \mathbb{N}$.

Let $f: Z^a \rightarrow Z$.

Show there exists a $D \subseteq Z$ such that

f restricted to D^a is not onto.

Let $g(a)$ be the number of times your proof uses the Ramsey's theorem.

If you did the problem correctly then $g(a)$ does not have a nice closed form but it is the solution to a combinatorial problem. What is that problem?

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4. (30 points) **Recall** that a finite set X is *Large* if $|X| > \min(X)$.

Prove the following:

For all k , there exists an n , such that, for all $\text{COL}: \binom{\{k, k+1, \dots, k+n\}}{2} \rightarrow [\omega]$ there is a large set that is either

- *homog*
- *min-homog*
- *max-homog*
- *rainbow*

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5. (0 points, but you must answer it)

As you all know, Yasmine's stage name is *Bad Sequence*.

If I had a rock band I would call it *The Red Cliques*.

A grad student in quantum computing had a group called *Amplitudes with Attitude*.

Dave Mount (Geometry) would name his group *Fried Green Ellipsoids*.

The following are real groups:

The Mobius Band,

The Klein Four,

The MIT Logarithms.

There is a rap singer named MC++. There is subgenre of rap called *Nerdcore* where people sing about math and technology things. *Drama in the PhD* is the best example of that. Do not confuse this with *mathcore* which is a *highly technical, chaotic subgenre of hardcore punk and metalcore* which in my opinion is just noise.

OKAY, so finally here is the question:

If you had a stage name, what would it be?

If you had a rock group, what would you name it?