Homework 09, Morally Due 12:30PM, Tue Apr 14 2026

1. (0 points) What is your name?

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2. (30 points) **Recall** The infinite 3-ary Can Ramsey Theorem: For all $\mathrm{COL}\binom{\mathsf{N}}{3}\to[\omega]$ there is an infinite $H \subseteq \mathsf{N}$ such that one of the following 8 cases occurs.

   (If we write $\mathrm{COL}(x, y, z)$ then $x < y < z$.)

   - A $\emptyset$-homog set, so all triples are the same color.
   - A $\{1\}$-homog set, so $\mathrm{COL}(x_1, x_2, x_3) = \mathrm{COL}(y_1, y_2, y_3)$ iff $x_1 = y_1$.
   - A $\{2\}$-homog set, so $\mathrm{COL}(x_1, x_2, x_3) = \mathrm{COL}(y_1, y_2, y_3)$ iff $x_2 = y_2$.
   - A $\{3\}$-homog set, so $\mathrm{COL}(x_1, x_2, x_3) = \mathrm{COL}(y_1, y_2, y_3)$ iff $x_3 = y_3$.
   - A $\{1, 2\}$-homog set, so $\mathrm{COL}(x_1, x_2, x_3) = \mathrm{COL}(y_1, y_2, y_3)$ iff ($x_1 = y_1$ and $x_2 = y_2$).
   - A $\{1, 3\}$-homog set, so $\mathrm{COL}(x_1, x_2, x_3) = \mathrm{COL}(y_1, y_2, y_3)$ iff ($x_1 = y_1$ and $x_3 = y_3$).
   - A $\{2, 3\}$-homog set, so $\mathrm{COL}(x_1, x_2, x_3) = \mathrm{COL}(y_1, y_2, y_3)$ iff ($x_2 = y_2$ and $x_3 = y_3$).
   - A $\{1, 2, 3\}$-homog set, so $\mathrm{COL}(x_1, x_2, x_3) = \mathrm{COL}(y_1, y_2, y_3)$ iff ($x_1 = y_1$ and $x_2 = y_2$ and $x_3 = y_3$) (this is rainbow).

   If B is a set of three points in the plane then $\mathrm{AREA}(B)$ is the area of the triangle formed by the three points in B.

   Let X be an infinite set of points in the plane, no three colinear.

   Show that there is an infinite subset H of X such that the map

   $B \in \binom{H}{3}$ goes to $\mathrm{AREA}(B)$ is injective

   (So all triangles have a different area.)

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3. (30 points) Let $n \in \mathsf{N}$ and $n \geq 3$. Find the constant $\alpha, \beta \in \mathsf{Q}$ such that the following is true, and prove it:

   If $n \equiv 0 \pmod 2$ and COL: $\binom{[n]}{2} \rightarrow [2]$ then there are at least $\frac{n^3}{24} + \alpha n + \beta$ mono K_3's.

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4. (40 points) In this problem we guide you through another proof of the Happy ending theorem.

For all $k \geq 3$, there exists an n, such that, for set X of n points in the plane there exists $Y \subseteq X$ with $|Y| = k$, and the points in Y are the vertices of a convex hull of size k.

We show that $n = R_3(k)$ suffices (for all COL: $\binom{[n]}{3} \rightarrow [2]$ there is a homog set of size k).

Let $X = \{p_1, \ldots, p_n\}$.

Let COL: $\binom{[n]}{3}$ be defined as follows:

COL($i < j < k$) =

- RED if p_i-p_j-p_k is CLOCKWISE
- BLUE if p_i-p_j-p_k is COUNTER-CLOCKWISE

(Do you young folk even know what CLOCKWISE means, with your fancy digital watches?)

FINISH THE PROOF.

5. (0 points but you must do it.) Heya, Javier here. Bill allowed me to add a 0 point question so here it goes. This one is particularly close to my heart, so have fun with it.

   - If you were an animal, which species would it be? No restrictions, you can use whatever criteria you like.
   - Why did you choose that species in particular?