752 Final for Spring 2026

1. (0 points) What is your name

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2. (20 points) Recall that $W(k, 2)$ is

   the least n *such that, for all* $\mathrm{COL}\colon [n] \rightarrow [2]$*, there is a mono* k*-AP.*

   Use the probabilistic method to obtain a lower bound on $W(k, 2)$.

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3. (20 points) Let ACK(n, m) be Ackermann's function. Let $f(n) = \text{ACK}(n, n)$.

If X is a finite subset of N then $\min(X)$ is the smallest element of X.
Hence $f(\min(X))$ is f applied to the smallest element of X.

Def Let X be a finite subset of N. X is *superlarge* if $|X| > f(\min(X))$.
Prove the following:
For every $k \in \mathsf{N}$ there exists n such that,
for every COL: $\binom{k,k+1,\ldots,k+n}{3}$ *there exists a superlarge homog set.*

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4. (20 points) Let E be the equation

$$x_1 + 2x_2 + 4x_3 - 8x_4 = 0$$

Give a finite coloring of N that has no mono solution in N. Prove your coloring has no mono solution. (For this problem 0 is NOT an element of N.)

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5. (20 points) Recall that infinite a-ary Ramsey Theorem:

For all $a \in \mathsf{N}$, *for all* $\mathrm{COL}\colon \binom{\mathsf{N}}{a} \rightarrow [2]$ *there exists an infinite homog set.*

We denote the above statement as a-ary IRT (Infinite Ramsey Theorem).

Assume that you already know the 1-ary IRT, 2-ary IRT, $\ldots$, 99-ary IRT.

Prove the 100-ary IRT.

(Hint: There are many different proofs depending on which a-IRT you want to use infinitely often and which one at the end. Do the easiest one which is NOT the one that gives the best bounds when made finite.)

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